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Vol 9 of 10

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Preface to the Series

For almost a decade, we have been receiving tremendous responses from GATE aspirants for our earlier books: GATE Multiple Choice Questions, GATE Guide, and the GATE Cloud series. Our first book, GATE Multiple Choice Questions (MCQ), was a compilation of objective questions and solutions for all subjects of GATE Electronics & Communication Engineering in one book. The idea behind the book was that Gate aspirants who had just completed or about to finish their last semester to achieve his or her B.E/B.Tech need only to practice answering questions to crack GATE. The solutions in the book were presented in such a manner that a student needs to know fundamental concepts to understand them. We assumed that students have learned enough of the fundamentals by his or her graduation. The book was a great success, but still there were a large ratio of aspirants who needed more preparatory materials beyond just problems and solutions. This large ratio mainly included average students.

Later, we perceived that many aspirants couldn't develop a good problem solving approach in their B.E/B.Tech. Some of them lacked the fundamentals of a subject and had difficulty understanding simple solutions. Now, we have an idea to enhance our content and present two separate books for each subject: one for theory, which contains brief theory, problem solving methods, fundamental concepts, and points-to-remember. The second book is about problems, including a vast collection of problems with descriptive and step-by-step solutions that can be understood by an average student. This was the origin of *GATE Guide* (the theory book) and *GATE Cloud* (the problem bank) series: two books for each subject. *GATE Guide* and *GATE Cloud* were published in three subjects only.

Thereafter we received an immense number of emails from our readers looking for a complete study package for all subjects and a book that combines both *GATE Guide* and *GATE Cloud*. This encouraged us to present GATE Study Package (a set of 10 books: one for each subject) for GATE Electronic and Communication Engineering. Each book in this package is adequate for the purpose of qualifying GATE for an average student. Each book contains brief theory, fundamental concepts, problem solving methodology, summary of formulae, and a solved question bank. The question bank has three exercises for each chapter: 1) Theoretical MCQs, 2) Numerical MCQs, and 3) Numerical Type Questions (based on the new GATE pattern). Solutions are presented in a descriptive and step-by-step manner, which are easy to understand for all aspirants.

We believe that each book of GATE Study Package helps a student learn fundamental concepts and develop problem solving skills for a subject, which are key essentials to crack GATE. Although we have put a vigorous effort in preparing this book, some errors may have crept in. We shall appreciate and greatly acknowledge all constructive comments, criticisms, and suggestions from the users of this book. You may write to us at rajkumar.kanodia@gmail.com and ashish.murolia@gmail.com.

Acknowledgements

We would like to express our sincere thanks to all the co-authors, editors, and reviewers for their efforts in making this project successful. We would also like to thank Team NODIA for providing professional support for this project through all phases of its development. At last, we express our gratitude to God and our Family for providing moral support and motivation.

We wish you good luck !

R. K. Kanodia
Ashish Murolia

SYLLABUS

GATE Electronics & Communications:

Random signals and noise: probability, random variables, probability density function, autocorrelation, power spectral density. Analog communication systems: amplitude and angle modulation and demodulation systems, spectral analysis of these operations, superheterodyne receivers; elements of hardware, realizations of analog communication systems; signal-to-noise ratio (SNR) calculations for amplitude modulation (AM) and frequency modulation (FM) for low noise conditions. Fundamentals of information theory and channel capacity theorem. Digital communication systems: pulse code modulation (PCM), differential pulse code modulation (DPCM), digital modulation schemes: amplitude, phase and frequency shift keying schemes (ASK, PSK, FSK), matched filter receivers, bandwidth consideration and probability of error calculations for these schemes. Basics of TDMA, FDMA and CDMA and GSM.

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IES Electronics & Telecommunication

Communication Systems: Basic information theory; Modulation and detection in analogue and digital systems; Sampling and data reconstructions; Quantization & coding; Time division and frequency division multiplexing; Equalization; Optical Communication: In free space & fiber optic; Propagation of signals at HF, VHF, UHF and microwave frequency; Satellite Communication.

IES Electrical

Communication Systems Types of modulation; AM, FM and PM. Demodulators. Noise and bandwidth considerations. Digital communication systems. Pulse code modulation and demodulation. Elements of sound and vision broadcasting. Carrier communication. Frequency division and time division multiplexing, Telemetry system in power engineering.

CONTENTS

CHAPTER 6

DIGITAL TRANSMISSION

6.1 INTRODUCTION

In this chapter, we will consider the digital transmission of analog messages via pulse modulation. This chapter covers the following topics:

- Sampling, which is basic to digital signal processing and digital communications.
- Pulse amplitude modulation (PAM), which is the simplest form of pulse modulation.
- Quantization, which represents an analog signal in discrete form in both amplitude and time.
- Pulse code modulation (PCM): standard method for the transmission of an analog message signal by digital means.
- Delta modulation (DM), which provides a staircase approximation to the oversampled version of the message signal.
- Digital signaling, which provides the mathematical representation for a digital signal waveform.
- Multiplexing, which enables the joint utilization of common channel by a plurality of independent message sources without mutual interference among them.

6.2 SAMPLING PROCESS

The sampling process is usually described in the time domain. In this process, an analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time. Consider an arbitrary signal $x(t)$ of finite energy, which is specified for all time as shown in figure 6.1(a).

Suppose that we sample the signal $x(t)$ instantaneously and at a uniform rate, once every T_s seconds, as shown in figure 6.1(b). Consequently, we obtain an infinite sequence of samples spaced T_s seconds apart and denoted by $\{x(nT_s)\}$, where n takes on all possible integer values.

Thus, we define the following terms:

1. **Sampling Period:** The time interval between two consecutive samples is referred as sampling period. In figure 6.1(b), T_s is the sampling period.

2. **Sampling Rate:** The reciprocal of sampling period is referred as sampling rate, i.e.

$$f_s = 1/T_s$$

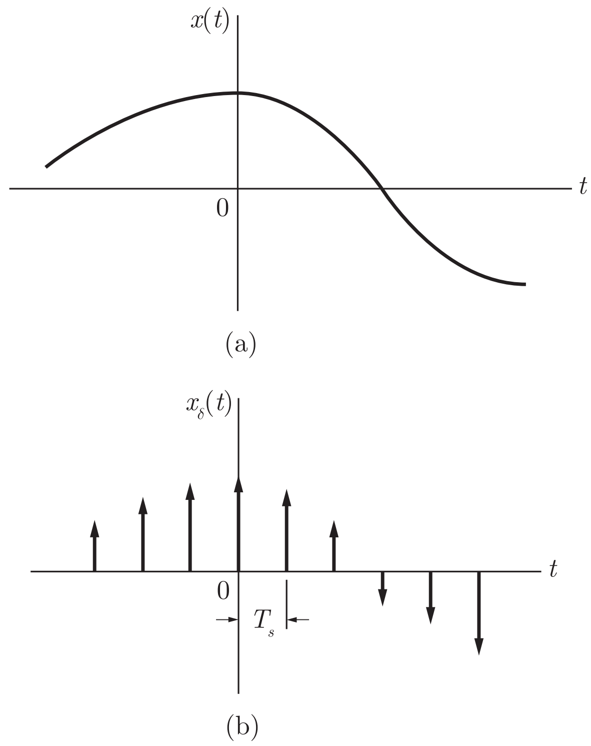


Figure 6.1: Illustration of Sampling Process: (a) Message Signal, (b) Sampled Signal

6.2.1 Sampling Theorem

Sampling theorem provides both a method of reconstruction of the original signal from the sampled values and also gives a precise upper bound on the sampling interval required for distortionless reconstruction. It states that

1. A band-limited signal of finite energy, which has no frequency components higher than W Hertz, is completely described by specifying the values of the signal at instants of time separated by $1/2\,W$ seconds.
2. A band-limited signal of finite energy, which has no frequency components higher that W Hertz, may be completely recovered from a knowledge of its samples taken at the rate of $2\,W$ samples per second.

6.2.2 Explanation of Sampling Theorem

Consider a message signal $m(t)$ bandlimited to W, i.e.

$$M(f) = 0 \qquad \text{for } |f| \geq W$$

Then, the sampling Frequency f_s, required to reconstruct the bandlimited waveform without any error, is given by

$$f_s \geq 2W$$

6.2.3 Nyquist Rate

Nyquist rate is defined as the minimum sampling frequency allowed to reconstruct a bandlimited waveform without error, i.e.

$$f_N = \min\{f_s\} = 2W$$

where W is the message signal bandwidth, and f_s is the sampling frequency.

6.2.4 Nyquist Interval

The reciprocal of Nyquist rate is called the Nyquist interval (measured in seconds), i.e.

$$T_N = \frac{1}{f_N} = \frac{1}{2W}$$

where f_N is the Nyquist rate, and W is the message signal bandwidth.

6.3 PULSE MODULATION

Pulse modulation is the process of changing a binary pulse signal to represent the information to be transmitted. Pulse modulation can be either analog or digital.

6.3.1 Analog Pulse Modulation

Analog pulse modulation results when some attribute of a pulse varies continuously in one-to-one correspondence with a sample value. In analog pulse modulation systems, the amplitude, width, or position of a pulse can vary over a continuous range in accordance with the message amplitude at the sampling instant, as shown in Figure 6.2. These lead to the following three types of pulse modulation:

1. Pulse Amplitude Modulation (PAM)
2. Pulse Width Modulation (PWM)
3. Pulse Position Modulation (PPM)

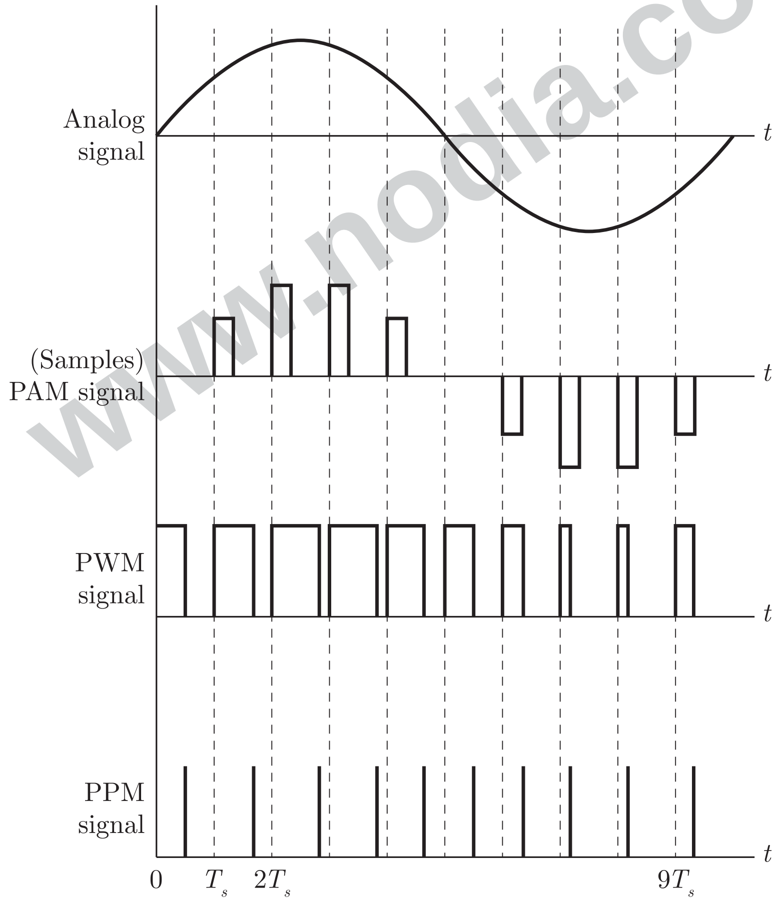


Figure 6.2: Representation of Various Analog Pulse Modulation

6.3.2 Digital Pulse Modulation

In systems utilizing digital pulse modulation, the transmitted samples take on only discrete values. Two important types of digital pulse modulation are:

1. Delta Modulation (DM)
2. Pulse Code Modulation (PCM)

In the following sections, we will discuss the various types of pulse modulation in some more detail.

6.4 PULSE AMPLITUDE MODULATION

Pulse amplitude modulation (PAM) is the conversion of the analog signal to a pulse-type signal in which the amplitude of the pulse denotes the analog information. PAM system utilizes two types of sampling: (1) Natural sampling and (2) Flat-top sampling.

6.4.1 Natural Sampling (Gating)

Consider an analog waveform $m(t)$ bandlimited to W hertz, as shown in Figure 6.3(a). The PAM signal that uses natural sampling (gating) is defined as

$$m_s(t) = m(t)s(t)$$

where $s(t)$ is the pulse waveform shown in Figure 6.3(b), and $m_s(t)$ is the resulting PAM signal shown in Figure 6.3(c)

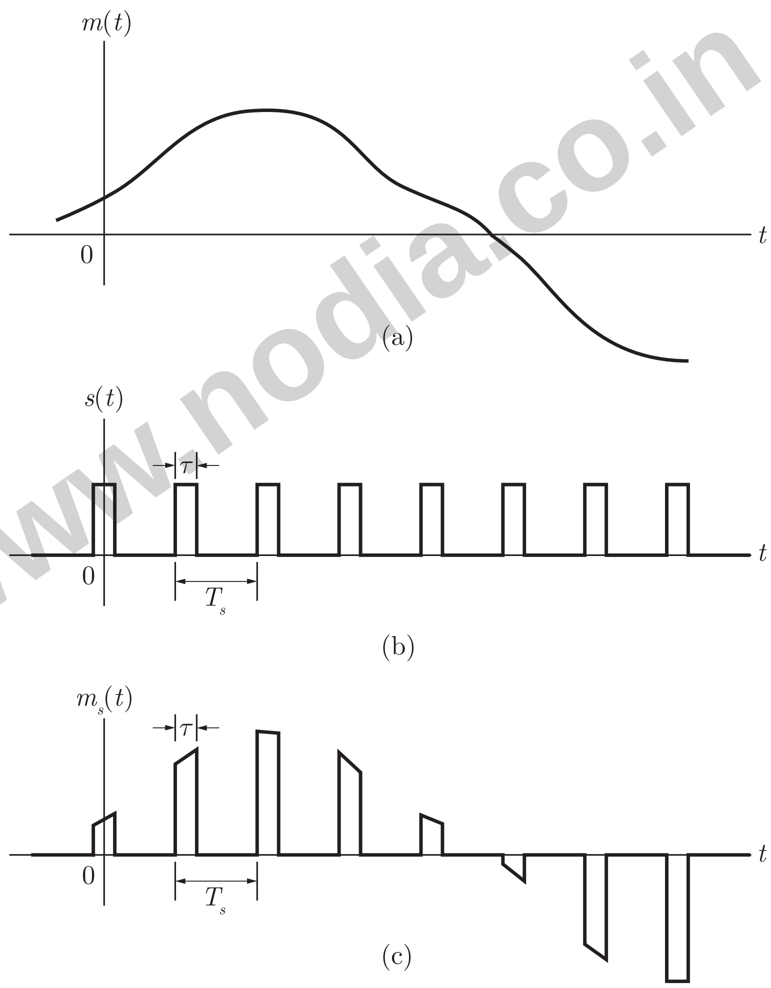


Figure 6.3: Illustration of Nautural Sampling Pulse Amplitude Modulation: (a) Message Signal, (b) Pulse Waveform, (c) Resulting PAM Signal

6.4.2 Instantaneous Sampling (Flat-Top PAM)

Analog waveforms may also be converted to pulse signalling by the use of flat-top signalling with instantaneous sampling, as shown in Figure 6.4. If $m(t)$ is an analog waveform bandlimited to W hertz, the instantaneous sampled PAM signal is given by

$$m_s(t) = \sum_{k=-\infty}^{\infty} m(kT_s)h(t-kT_s)$$

where $h(t)$ denotes the sampling-pulse shape shown in Figure 6.4(b), and $m_s(t)$ is the resulting flat top PAM signal shown in Figure 6.4(c)

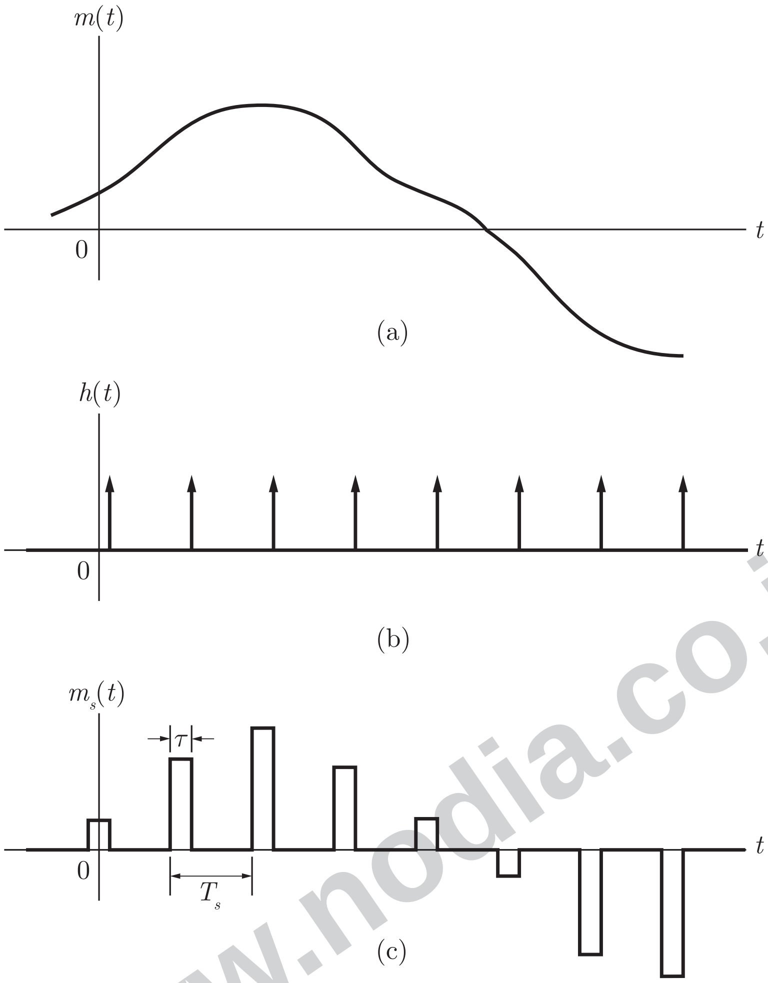


Figure 6.3: Illustration of Flat-top Sampling Pulse Amplitude Modulation: (a) Message Signal, (b) Sampling Pulse, (c) Resulting PAM Signal

POINT TO REMEMBER

The analog-to-PAM conversion process is the first step in converting an analog waveform to a PCM (digital) signal.

6.5 PULSE CODE MODULATION

Pulse code modulation (PCM) is essentially analog-to-digital conversion of a special type where the information contained in the instantaneous samples of an analog signal is represented by digital words in a serial bit stream. Figure 6.5 shows the basic elements of a PCM system. The PCM signal is generated by carrying out the following three basic operations:

1. Sampling
2. Quantizing
3. Encoding

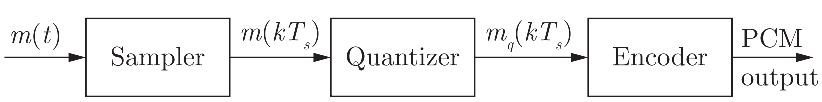


Figure 6.4 : Block Diagram Representation of PCM System

6.5.1 Sampling

The incoming message signal $m(t)$ is sampled with a train of narrow rectangular pulses so as to closely approximate the instantaneous sampling process. To ensure perfect reconstruction of the message signal at the receiver, the sampling rate must be greater than twice the highest frequency component W of the message signal in accordance with the sampling theorem. The resulting sampled waveform $m(kT_s)$ is discrete in time.

APPLICATION OF SAMPLING

The application of sampling permits the reduction of the continuously varying message signal (of some finite duration) to a limited number of discrete values per second.

6.5.2 Quantization

A quantizer rounds off the sample values to the nearest discrete value in a set of q quantum levels. The resulting quantized samples $m_q(kT_s)$ are discrete in time (by virtue of sampling) and discrete in amplitude (by virtue of quantizing). Basically, quantizers can be of a uniform or nonuniform type.

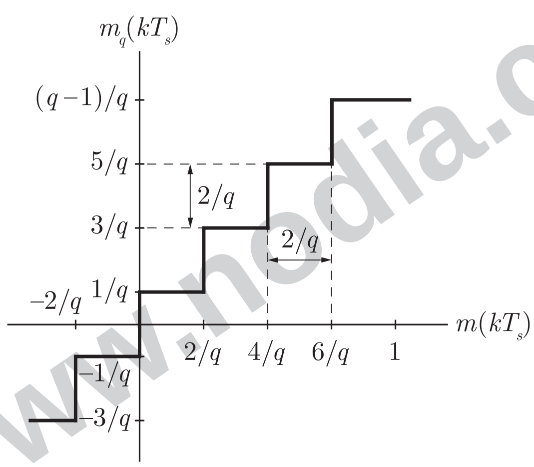


Figure 6.6: Representation of Relationship Between Sampled and Quantized Signal

Uniform Quantizer

A quantizer is called as a uniform quantizer if the step size remains constant throughout the input range. To display the relationship between $m(kT_s)$ and $m_q(kT_s)$, let the analog message be a voltage waveform normalized such that $|m(t)| \le 1\,V$. Uniform quantization subdivides the 2 V peak-to-peak range into q equal steps of height $2/q$, as shown in Figure 6.6. The quantum levels are then taken to be at $\pm 1/q$, $\pm 3/q$,..., $\pm (q-1)/q$ in the usual case when q is an even integer. A quantized value such as $m_q(kT_s) = 5/q$ corresponds to any sample value in the range $4/q < x(xT_s) < 6/q$.

Nonuniform Quantizer

Nonuniform quantization is required to be implemented to improve the signal to quantization noise ratio of weak signals. It is equivalent to passing the baseband signal through a compressor and then applying the compressed signal to a uniform quantizer. A particular form of compression law that is used in practice is the so called μ-law, which is defined by

$$|m_q| = \frac{\ln(1+\mu|m|)}{\ln(1+\mu)}$$

where m and m_q are the normalized input and output voltages, and μ is a positive constant.

6.5.3 Encoding

An encoder translates the quantized samples into digital code words. The encoder works with M-ary digits and produces for each sample a code word consisting of n digits in parallel. Since, there are M^n possible M-ary codewords with n digits per word, unique encoding of the q different quantum levels requires that

$$M^n \geq q$$

The parameters M, n, and q should be chosen to satisfy the equality, so that

$$q = M^n \quad \text{or} \quad n = \log_M q$$

Encoding in Binary PCM

For binary PCM, each digit may be either of two distinct values 0 or 1, i.e.

$$M = 2$$

If the code word of binary PCM consists of n digits, then number of quantization levels is defined as

$$q = 2^n$$

or $$n = \log_2 q$$

In general, we must remember the following characteristics of a PCM system:

CHARACTERISTICS OF PCM SYSTEM

1. A sampled waveform is quantized into q quantization levels; where q is an integer.
2. If the message signal is defined in the range $(-m_p, m_p)$, then the step size of quantizer is
$$\delta = \frac{2m_p}{q}$$
3. For a binary PCM system with n digit codes, the number of quantization level is defined as
$$q = 2^n$$
4. If the message signal is sampled at the sampling rate f_s, and encoded to n number of bits per sample; then the bit rate (bits/sec) of the PCM is defined as
$$R_b = nf_s$$

METHODOLOGY 1 : TO EVALUATE BIT RATE FOR PCM SYSTEM

If the number of quantization levels q and message signal frequency f_m for a PCM signal is given, then bit rate for the PCM system is obtained in the following steps:

Step 1: Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by

$$f_s = 2f_m$$

Step 2: Deduce the number of bits per sample using the expression

$$n = \log_2 q$$

Step 3: Evaluate bit rate (bits/sec) for the PCM system by substituting the obtained values in the expression

$$R_b = nf_s$$

6.6 TRANSMISSION BANDWIDTH IN A PCM SYSTEM

The bandwidth of (serial) binary PCM waveforms depends on the bit rate and the waveform pulse shape used to represent the data. The dimensionality theorem shows that the bandwidth of the binary encoded PCM waveform is bounded by

$$B_{PCM} \geq \frac{1}{2}R_b = \frac{1}{2}nf_s$$

where R_b is the bit rate, n is the number of bits in PCM word, and f_s is the sampling rate. Since, the required sampling rate for no aliasing is

$$f_s \geq 2W$$

where W is the bandwidth of the message signal (that is to be converted to the PCM signal). Thus, the bandwidth of the PCM signal has a lower bound given by

$$B_{PCM} \geq nW$$

POINT TO REMEMBER

The minimum bandwidth of $\frac{1}{2}R = \frac{1}{2}nf_s$ is obtained only when $(\sin x)/x$ type pulse shape is used to generate the PCM waveform. However, usually a more rectangular type of pulse shape is used, and consequently, the bandwidth of the binary-encoded PCM waveform will be larger than this minimum. Thus, for rectangular pulses, the first null bandwidth is

$$B_{PCM} = R = nf_s \text{ (first null bandwidth)}$$

6.7 NOISE CONSIDERATION IN PCM

In PCM (pulse code modulation), there are two error sources:

1. Quantization noise
2. Channel noise

6.7.1 Quantization Noise

For a PCM system, the kth sample of quantized message signal is represented by

$$m_q(kT_s) = m(kT_s) + \varepsilon(kT_s)$$

where $m(kT_s)$ is the sampled waveform, and $\varepsilon(kT_s)$ is the quantization error. Let the quantization levels having uniform step size δ. Then, we have

$$-\frac{\delta}{2} \leq \varepsilon \leq \frac{\delta}{2}$$

So, the mean-square error due to quantization is

$$\overline{\varepsilon^2} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} \qquad ...(6.1)$$

METHODOLOGY 2 : TO EVALUATE BIT RATE FOR PCM SYSTEM

For a PCM system, consider the message signal having frequency f_m and peak to peak amplitude $2m_p$. If the accuracy of the PCM system is given as $\pm x$ % of full scale value, then the bit rate is obtained in the following steps:

Step 1 : Obtain the sampling frequency for the PCM signal. According to Nyquist criterion, the minimum sampling frequency is given by

$$f_s = 2f_m$$

Step 2 : Obtain the maximum quantization error for the PCM system using the expression

$$|\text{error}| = \left|\frac{\delta}{2}\right| = \left|\frac{2m_p}{2q}\right| = \left|\frac{m_p}{q}\right| = \left|\frac{m_p}{2^n}\right|$$

Step 3 : Apply the given condition of accuracy as

$$|\text{error}| \le x\% \text{ of full scale value}$$

Step 4 : Solve the above condition for the minimum value of number of bits per second (n).

Step 5 : Obtain the bit rate by substituting the approximated integer value of n in the expression

$$R_b = nf_s$$

6.7.2 Signal to Quantization Noise Ratio

For PCM system, we have the message signal $m(t)$, and quantization error ε. So, we define the signal to quantization noise ratio as

$$(\text{SNR})_Q = \frac{\overline{m^2(t)}}{\overline{\varepsilon^2}} = \frac{\overline{m^2(t)}}{\delta^2/12} \qquad ...(6.2)$$

where δ is the step size of the quantized signal defined as

$$\delta = \frac{2m_p}{q} \qquad ...(6.3)$$

Substituting equation (6.3) in equation (6.2), we get the expression for signal to quantization noise ratio as

$$(\text{SNR})_Q = 12\frac{\overline{m^2(t)}}{(2m_p/q)^2}$$

$$(\text{SNR})_Q = 3q^2\frac{\overline{m^2(t)}}{m_p^2} \qquad ...(6.4)$$

where m_p is the peak amplitude of message signal $m(t)$, and q is the number of quantization level. Let us obtain the more generalized form of SNR for the following two cases:

Case I :

When $m(t)$ is a sinusoidal signal, we have its mean square value

$$\overline{m^2(t)} = \frac{1}{2}$$

and the peak amplitude of sinusoidal message signal is

$$m_p = 1$$

So, by substituting these values in equation (6.4), we get the signal to quantization noise ratio for sinusoidal message signal as

$$(\text{SNR})_Q = 3q^2 \frac{1/2}{(1)^2} = \frac{3q^2}{2}$$

Case II :

When $m(t)$ is uniformly distributed in the range $(-m_p, m_p)$, then we obtain

$$\overline{m^2(t)} = \frac{m_p^2}{3}$$

Substituting this value in equation (6.4), we get the signal to quantization noise ratio as

$$(\text{SNR})_Q = 3q^2 \frac{m_p^2/3}{m_p^2} = q^2$$

Case III:

For any arbitrary message signal $m(t)$, the peak signal to quantization noise ratio is defined as

$$(\text{SNR})_{\text{peak}} = 3q^2 \frac{m_p^2}{m_p^2} = 3q^2$$

6.7.3 Channel Noise

If a PCM signal is composed of the data that are transmitted over the channel having bit error rate P_e, then peak signal to average quantization noise ratio is defined as

$$(\text{SNR})_{\text{peak}} = \frac{3q^2}{1 + 4(q^2-1)P_e}$$

Similarly, for the channel with bit error probability P_e, the average signal to average quantization noise ratio is defined as

$$(\text{SNR})_{\text{ave}} = \frac{q^2}{1 + 4(q^2-1)P_e}$$

POINT TO REMEMBER

If the additive noise in the channel is so small that errors can be neglected, quantization is the only error source in PCM system.

6.7.4 Companding

Companding is nonuniform quantization. It is required to be implemented to improve the signal to quantization noise ratio of weak signals. The signal to quantization noise ratio for μ-law companding is approximated as

$$(\text{SNR})_Q = \frac{3q^2}{[\ln(1+\mu)]^2}$$

where q is the number of quantization level, and μ is a positive constant.

6.8 ADVANTAGES OF PCM SYSTEM

PCM is very popular because of the many advantages it offers, including the following:

1. Relatively inexpensive digital circuitry may be used extensively in the system.
2. PCM signals derived from all types of analog sources (audio, video, etc.) may be merged with data signals (e.g., from digital computers) and transmitted over a common high-speed digital communication system.

3. In long-distance digital telephone systems requiring repeaters, a clean PCM waveform can be regenerated at the output of each repeater, where the input consists of a noisy PCM waveform.
4. The noise performance of a digital system can be superior to that of an analog system.

POINT TO REMEMBER

The advantages of PCM usually outweigh the main disadvantage of PCM: a much wider bandwidth than that of the corresponding analog signal.

6.9 DELTA MODULATION

Delta modulation provides a staircase approximation to the oversampled version of the message signal, as illustrated in Figure 6.7. Let $m(t)$ denote the input (message) signal, and $m_q(t)$ denote its staircase approximation. The difference between the input and the approximation is quantized into only two levels, namely, $\pm\delta$, corresponding to positive and negative differences. Thus if the approximation falls below the signal at any sampling approach, it is increased by δ. If on the other hand, the approximation lies above the signal, it is diminished by δ.

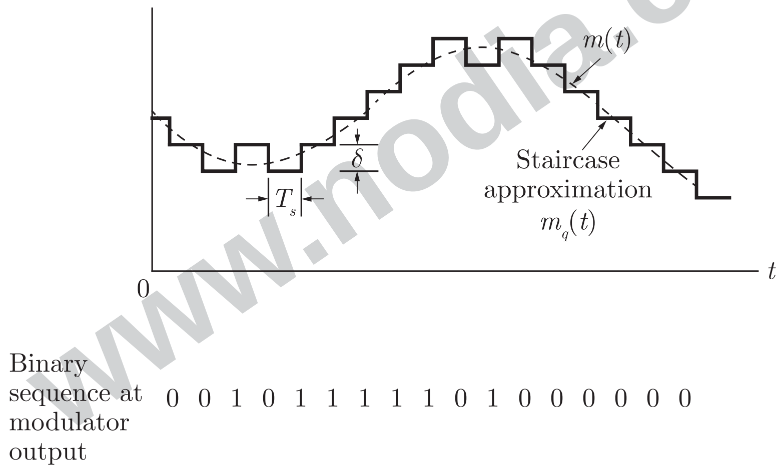


Figure 6.7: Staircase Approximation in Delta Modulation

Following are some key points related to delta modulation.

POINTS TO REMEMBER

1. In delta modulation (DM), an incoming message signal is oversampled (i.e. at a rate much higher than the Nyquist rate) to purposely increase the correlation between adjacent samples of the signal
2. The staircase approximation remains within $\pm\delta$ of the input signal provided that the signal does not change too rapidly from sample to sample.

6.9.1 Noise Consideration in Delta Modulation

The quantizing noise error in delta modulation can be classified into two types of noise:

1. Slope Overload Noise
2. Granular Noise

1. Slope Overload Noise

Slope overload noise occurs when the step size δ is too small for the accumulator output to follow quick changes in the input waveform. The maximum slope that can be generated by the accumulator output is

$$\frac{\delta}{T_s} = \delta f_s$$

where T_s is sampling interval, and f_s is the sampling rate. To prevent the slope overload noise, the maximum slope of the message signal must be less than maximum slope generated by accumulator. Thus, we have the required condition to avoid slope overload as

$$\max\left|\frac{dm(t)}{dt}\right| \le \delta f_s$$

where $m(t)$ is the message signal, δ is the step size of quantized signal, and f_s is the sampling rate.

2. Granular Noise

The granular noise in a DM system is similar to the granular noise in a PCM system. From equation (6.1), we have the total quantizing noise for PCM system,

$$(\overline{\varepsilon^2})_{\text{PCM}} = \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} \varepsilon^2 d\varepsilon = \frac{\delta^2}{12} = \frac{(\delta/2)^2}{3}$$

Replacing $\delta/2$ of PCM by δ for DM, we obtain the total granular quantizing noise as

$$(\overline{\varepsilon^2})_{\text{DM}} = \frac{\delta^2}{3}$$

Thus, the power spectral density for granular noise in delta modulation system is obtained as

$$S_N(f) = \frac{\delta^2/3}{2f_s} = \frac{\delta^2}{6f_s}$$

where δ is the step size, and f_s is the sampling frequency.

POINT TO REMEMBER

Granular noise occurs for any step size but is smaller for a small step size. Thus we would like to have δ as small as possible to minimize the granular noise.

METHODOLOGY : MINIMUM STEP SIZE IN DELTA MODULATION

Following are the steps involved in determination of minimum step size to avoid slope overload in delta modulation:

Step 1: Obtain the sampling frequency for the modulation. According to Nyquist criterion, the minimum sampling frequency is given by

$$f_s = 2f_m$$

Step 2: Obtain the maximum slope of message signal using the expression

$$\max\left|\frac{dm(t)}{dt}\right| = 2\pi f_m A_m$$

where f_m is the message signal frequency, and A_m is amplitude of the message signal.

Step 3: Apply the required condition to avoid slope overload as

$$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

Step 4: Evaluate the minimum value of step size δ by solving the above condition.

6.10 MULTILEVEL SIGNALING

In a multilevel signaling scheme, the information source emits a sequence of symbols from an alphabet that consists of M symbols (levels). Let us understand some important terms used in multilevel signaling.

6.10.1 Baud

Let a multilevel signaling scheme having the symbol duration T_s seconds. So, we define the symbols per second transmitted for the system as

$$D = \frac{1}{T_s}$$

where D is the symbol rate which is also called *baud*.

6.10.2 Bits per Symbol

For a multilevel signaling scheme with M number of symbols (levels), we define the bits per symbol as

$$k = \log_2 M$$

6.10.3 Relation Between Baud and Bit Rate

For a multilevel signaling scheme, the bit rate and baud (symbols per second) are related as

$$R_b = kD = D\log_2 M \qquad ...(6.5)$$

where R_b is the bit rate, $k = \log_2 M$ is the bits per symbol, and D is the baud (symbols per second).

6.10.4 Relation Between Bit Duration and Symbol Duration

For a multilevel signaling scheme, the bit duration is given by

$$T_b = \frac{1}{R_b}$$

where R_b is the bit rate. Also, we have the symbol duration

$$T_s = \frac{1}{D}$$

where D is the symbol rate. Thus, by substituting these expressions in equation (6.5), we get the relation

$$T_s = kT_b = T_b \log_2 M$$

where $k = \log_2 M$ is the bits per symbol.

6.10.5 Transmission Bandwidth

The null to null transmission bandwidth of the rectangular pulse multilevel waveform is defined as

$$B_T = D \text{ symbols/sec}$$

The absolute transmission bandwidth for $\frac{\sin x}{x}$ pulse multilevel waveform is defined as

$$B_T = \frac{D}{2} \text{ symbols/sec}$$

6.11 MULTIPLEXING

In many applications, a large number of data sources are located at a common point, and it is desirable to transmit these signals simultaneously using a single communication channel. This is accomplished using multiplexing. Let us study the two important types of multiplexing: FDM and TDM.

6.11.1 Frequency-Division Multiplexing (FDM)

Frequency-division multiplexing (FDM) is a technique whereby several message signals are translated, using modulation, to different spectral locations and added to form a baseband signal. The carriers used to form the baseband are usually referred to as subcarriers. Then, if desired, the baseband signal can be transmitted over a single channel using a single modulation process.

Bandwidth of FDM Baseband Signal

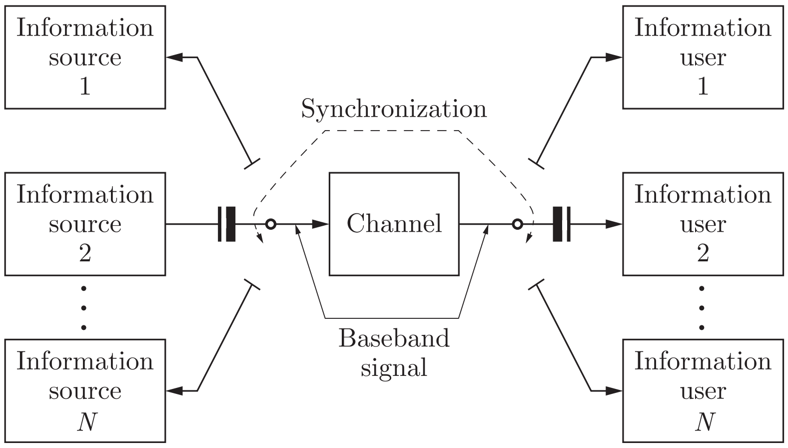
The bandwidth of FDM baseband is equal to the sum of the bandwidths of the modulated signals plus the sum of the guardbands, the empty spectral bands between the channels necessary for filtering. This bandwidth is lower-bounded by the sum of the bandwidths of the message signals, i.e.

$$B = \sum_{i=1}^{N} W_i$$

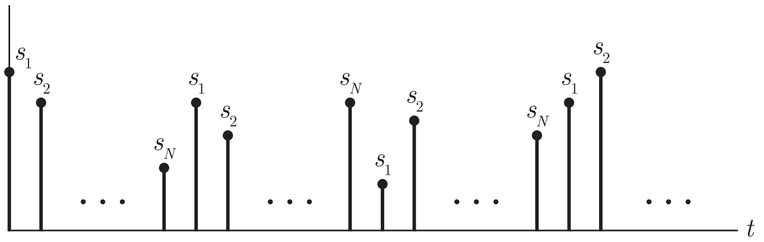
where W_i is the bandwidth of $m_i(t)$, This bandwidth is achieved when all baseband modulators are SSB and all guardbands have zero width.

6.11.2 Time Division Multiplexing (TDM)

Time-division multiplexing provides the time sharing of a common channel by a large number of users. Figure 6.8(a) illustrates a TDM system. The data sources are assumed to have been sampled at the Nyquist rate or higher. The commutator then interlaces the samples to form the baseband signal shown in Figure 6.8(b). At the channel output, the baseband signal is demultiplexed by using a second commutator as illustrated. Proper operation of this system depends on proper synchronization between the two commutators.



(a)



(b)

Figure 6.8: (a) Time Division Multiplexing System, (b) Resulting Baseband Signal

In a TDM system, the samples are transmitted depending on the message signal bandwidth. For example, let us consider the following two cases:

1. If all message signals have equal bandwidth, then the samples are transmitted sequentially, as shown in Figure 6.8(b).
2. If the sampled data signals have unequal bandwidths, more samples must be transmitted per unit time from the wideband channels. This is easily accomplished if the bandwidths are harmonically related. For example, assume that a TDM system has four data sources $s_1(t)$, $s_2(t)$, $s_3(t)$, and $s_4(t)$ having the bandwidths respectively as $W, W, 2W, 4W$. Then, it is easy to show that a permissible sequence of baseband samples is a periodic sequence, one period of which is $...s_1 s_4 s_3 s_4 s_2 s_4 ....$

Bandwidth of TDM Baseband Signal

The minimum sampling bandwidth of a TDM baseband signal is defined as

$$B = \sum_{i=1}^{N} W_i$$

where W_i is the bandwidth of the ith channel.

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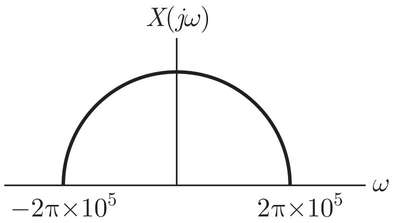
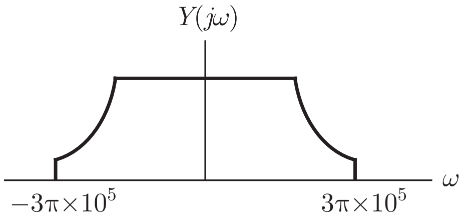
EXERCISE 6.1

Common Data For Q. 1 to 5 :

Figure given below shows Fourier spectra of signal $x(t)$ and $y(t)$.





MCQ 6.1.1 The Nyquist sampling rate for $x(t)$ is

(A) 100 kHz (B) 200 kHz
(C) 300 kHz (D) 50 kHz

MCQ 6.1.2 The Nyquist sampling rate for $y(t)$ is

(A) 50 kHz (B) 75 kHz
(C) 150 kHz (D) 300 kHz

MCQ 6.1.3 The Nyquist sampling rate for $x^2(t)$ is

(A) 100 kHz (B) 150 kHz
(C) 250 kHz (D) 400 kHz

MCQ 6.1.4 The Nyquist sampling rate for $y^3(t)$ is

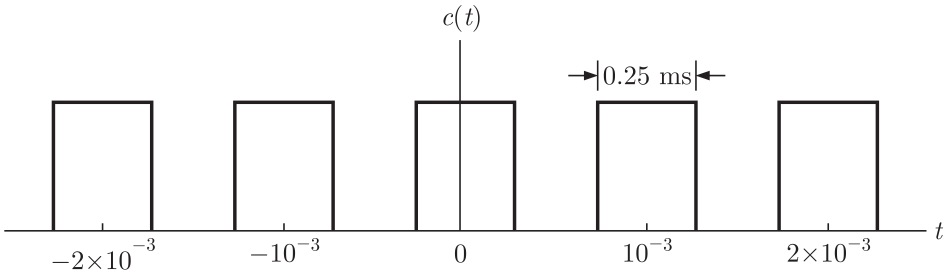
(A) 100 kHz (B) 300 kHz
(C) 900 kHz (D) 120 kHz

MCQ 6.1.5 The Nyquist sampling rate for $x(t)y(t)$ is

(A) 250 kHz (B) 500 kHz
(C) 50 kHz (D) 100 kHz

Common Data For Q. 6 and 7

A signal $x(t)$ is multiplied by rectangular pulse train $c(t)$ shown in figure.



MCQ 6.1.6 $x(t)$ would be recovered from the product $x(t)c(t)$ by using an ideal LPF if $X(j\omega)=0$ for

(A) $\omega > 2000\pi$ (B) $\omega > 1000\pi$
(C) $\omega < 1000\pi$ (D) $\omega < 2000\pi$

MCQ 6.1.7 If $X(j\omega)$ satisfies the constrains required, then the pass band gain A of the ideal lowpass filter needed to recover $x(t)$ from $c(t)x(t)$ is
(A) 1 (B) 2
(C) 4 (D) 8

Common Data For Q. 8 and 9 :

Ten telemetry signals, each of bandwidth 2 kHz, are to be transmitted simultaneously by binary PCM. The maximum tolerable error in sample amplitudes is 0.2% of the peak signal amplitude. The signals must be sampled at least 20% above the Nyquist rate. Framing and synchronizing requires an additional 1% extra bits.

MCQ 6.1.8 The minimum possible data rate must be
(A) 272.64 k bits/sec
(B) 436.32 k bits/sec
(C) 936.32 k bits/sec
(D) None of the above

MCQ 6.1.9 The minimum transmission bandwidth is
(A) 218.16 kHz (B) 468.32 kHz
(C) 136.32 kHz (D) None of the above

MCQ 6.1.10 A binary channel with capacity 36 k bits/sec is available for PCM voice transmission. If signal is band limited to 3.2 kHz, then the appropriate values of quantizing level and the sampling frequency will be, respectively
(A) 32, 3.6 kHz (B) 64, 7.2 kHz
(C) 64, 3.6 kHz (D) 32, 7.2 kHz

Common Data For Q. 11 to 13 :

Consider a linear DM system designed to accommodate analog message signals limited to bandwidth of 3.5 kHz. A sinusoidal test signal of amplitude $A_m = 1$ V and frequency $f_m = 800$ Hz is applied to system. The sampling rate of the system is 64 kHz.

MCQ 6.1.11 The minimum value of the step size to avoid slope overload is
(A) 240 mV (B) 120 mV
(C) 670 mV (D) 78.5 mV

MCQ 6.1.12 The granular-noise power would be
(A) 1.68×10^{-3} W (B) 2.86×10^{-4} W
(C) 2.48×10^{-3} W (D) 1.12×10^{-4} W

MCQ 6.1.13 The SNR will be
(A) 298 (B) 1.75×10^{-3}
(C) 4.46×10^{3} (D) 201

Common Data For Q. 14 and 15 :

A Signal has a bandwidth of 1 MHz. It is sampled at a rate 50% higher than the Nyquist rate and quantized into 256 level using a μ-law quantizer with $\mu = 255$.

MCQ 6.1.14 The signal-to-quantization-noise ratio is

(A) 34.91 dB (B) 38.06 dB

(C) 42.05 dB (D) 48.76 dB

MCQ 6.1.15 It was found that a sampling rate 20% above the rate would be adequate. So the maximum SNR, that can be realized without increasing the transmission bandwidth, would be

(A) 64.4 dB (B) 70.3 dB

(C) 50.1 dB (D) None of the above

MCQ 6.1.16 The input to a linear delta modulator having step-size $\delta = 0.628$ is a sine wave with frequency f_m and peak amplitude A_m. If the sampling frequency $f_s = 40$ kHz, the combination of the sine wave frequency and the peak amplitude, where slope overload will take place is

	A_m	f_m
(A)	0.3 V	8 kHz
(B)	1.5 V	4 kHz
(C)	1.5 V	2 kHz
(D)	3.0 V	1 kHz

MCQ 6.1.17 A sinusoidal signal is sampled at 8 kHz and is quantized using 8 bit uniform quantizer. If SNR_q be the quantization signal to noise ratio and R be the bit rate of PCM signal, which of the following provides the correct values of SNR_q and R ?

(A) $R = 32$ kbps, $\text{SNR}_q = 25.8$ dB

(B) $R = 64$ kbps, $\text{SNR}_q = 49.8$ dB

(C) $R = 64$ kbps, $\text{SNR}_q = 55.8$ dB

(D) $R = 32$ kbps, $\text{SNR}_q = 49.8$ dB

MCQ 6.1.18 A 1.0 kHz signal is flat-top sampled at the rate of 1800 samples/sec and the samples are applied to an ideal rectangular LPF with cut-off frequency of 1100 Hz. The output of the filter contains

(A) only 800 Hz component

(B) 800 and 900 Hz component

(C) 800 Hz and 1000 Hz components

(D) 800 Hz, 900 and 1000 Hz components

MCQ 6.1.19 A signal $x(t) = 100\cos(24\pi \times 10^3)t$ is ideally sampled with a sampling period of 50 μsec and then passed through an ideal lowpass filter with cutoff frequency of 15 kHz. Which of the following frequencies is/are present at the filter output

(A) 12 kHz only (B) 8 kHz only

(C) 12 kHz and 9 kHz (D) 12 kHz and 8 kHz

MCQ 6.1.20 The minimum sampling frequency (in samples/sec) required to reconstruct the following signal form its samples without distortion would be

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

(A) 2×10^3 Hz (B) 4×10^3 Hz
(C) 6×10^3 Hz (D) 8×10^3 Hz

MCQ 6.1.21 The minimum step-size required for a Delta-Modulator operating at 32 K samples/sec to track the signal (here $u(t)$ is the unit function)

$$m(t) = 125t[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

so that slope overload is avoided, would be
(A) 2^{-10} (B) 2^{-8}
(C) 2^{-6} (D) 2^{-4}

MCQ 6.1.22 If the number of bits in a PCM system is increased from n to $n+1$, the signal-to-quantization noise ratio will increase by a factor.
(A) $\frac{(n+1)}{n}$ (B) $\frac{(n+1)^2}{n^2}$
(C) 2 (D) 4

MCQ 6.1.23 In PCM system, if the quantization levels are increased form 2 to 8, the relative bandwidth requirement will.
(A) remain same
(B) be doubled
(C) be tripled
(D) become four times

Common Data For Q. 24 to 26 :

A singer's performance is to be recorded by sampling and storing the sample values. Assume that the highest frequency tone to be recorded is 15800 hertz.

MCQ 6.1.24 What is the minimum sampling frequency that can be used ?
(A) 3.16×10^4 Hz (B) 15.8×10^4 Hz
(C) 7.9×10^3 Hz (D) 6.32×10^4 Hz

MCQ 6.1.25 How many samples would be required to store a three minutes performance ?
(A) 1.053×10^4 samples
(B) 5.688×10^6 samples
(C) 1.756×10^6 samples
(D) 9.48×10^4 samples

MCQ 6.1.26 If each sample is quantized into 128 levels, how many binary digits (bits) would be required to store the three minutes performance ?
(A) 8.125×10^5 bits (B) 7.28×10^8 bits
(C) 3.98×10^7 bits (D) 5.688×10^6 bits

Common Data For Q. 27 and 28 :

A voltage of 1 volt rms is applied at the sending end of a telephone cable of length 1000 meters. The attenuation in the cable is 1 dB/m.

MCQ 6.1.27 What will be the rms voltage at receiving end of the cable ?

(A) 10^{50} volt
(B) 10^{-50} volt
(C) 10^{5} volt
(D) 10^{-5} volt

MCQ 6.1.28 It is desired to obtain a 1 volt rms signal at the receiving end. This can be accomplished by using repeaters. If the repeaters yield a maximum output of 1 volt and have a voltage gain of 100, then the number of repeaters needed is

(A) 100
(B) 25
(C) 5
(D) 40

Common Data For Q. 29 to 31 :

The probability density of a signal $X(t)$ is given by

$$f_{X(t)}(x) = \begin{cases} ke^{-|x|} & \text{for } |X| \le 4 \text{ volt} \\ 0 & \text{otherwise} \end{cases}$$

MCQ 6.1.29 If the signal $X(t)$ is quantized into 4 levels, what will be the step size of each level ?

(A) 2 volt
(B) 8 volt
(C) 32 volt
(D) 1 volt

MCQ 6.1.30 What will be the value of constant k ?

(A) 9.16×10^{-3}
(B) 0.4908
(C) 27.29
(D) 0.5093

MCQ 6.1.31 If the probability density function is not constant over each level, what will be the variance of the quantization error ?

(A) 0.1869
(B) 0.5093
(C) 0.7341
(D) 0.3739

MCQ 6.1.32 Consider the two message signals defined as

$$m_1(t) = \sin c(100t),\ m_2(t) = \sin c^2(100t)$$

what are the values of Nyquist rates f_{N1} and f_{N2} of the corresponding signals ?

	f_{N1}	f_{N2}
(A)	100	200
(B)	200	100
(C)	50	100
(D)	100	50

MCQ 6.1.33 In a binary system, the quantizing noise is not to exceed $\pm x$ % of the peak-to-peak analog level. The minimum number of bits in each PCM words needs to be

(A) $x/50$
(B) $50/x$
(C) $\log_2\left(\frac{50}{x}\right)$
(D) $\log_2\left(\frac{x}{50}\right)$

Common Data For Q. 34 and 35 :

The information in an analog voltage waveform is to be transmitted over a PCM system with a $\pm 0.1\%$ accuracy. The analog waveform has an absolute bandwidth of 100 Hz and an amplitude range of -10 V to $+10$ V.

MCQ 6.1.34 What is the minimum sampling rate needed for the waveform ?

(A) 102 Hz (B) 100 Hz
(C) 200 Hz (D) 50 Hz

MCQ 6.1.35 What is the minimum absolute channel bandwidth required for transmission of this PCM ?

(A) 1800 Hz (B) 900 Hz
(C) 450 Hz (D) 3600 Hz

MCQ 6.1.36 An 850 Mbyte hard disk is used to store PCM data. Suppose that a voice frequency signal is sampled at 8 k samples/sec and the encoded PCM is to have an average SNR of at least 30 dB. How many minutes of voice frequency conversation can be stored on the hard disk ?

(A) 2.833 minute (B) 170×10^3 minute
(C) 2833 minute (D) 1700 minute

Common Data For Q. 37 and 38 :

An analog signal with a bandwidth of 4.2 MHz is to be converted into binary PCM and transmitted over a channel. The peak signal quantizing noise ratio must be at least 55 dB.

MCQ 6.1.37 If there is no bit error and no inter symbol interferrence, what will be the number of quantizing steps needed ?

(A) 512 (B) 9
(C) 4 (D) 256

MCQ 6.1.38 What will be the channel bandwidth required if rectangular pulse shapes are used ?

(A) 75.6 MHz (B) 8.4 MHz
(C) 933.3 kHz (D) 17.4 MHz

Common Data For Q. 39 and 40 :

Compact disk (CD) players use 16 bit PCM, including one parity bit with 8 times oversampling of the analog signal. The analog signal bandwidth is 20 kHz.

MCQ 6.1.39 What is the null bandwidth of this PCM signal ?

(A) 640 MHz (B) 320 kHz
(C) 512 MHz (D) 512 MHz

MCQ 6.1.40 What will be the peak SNR for the signal ?

(A) 9.50 dB (B) 10.1 dB
(C) 95.08 dB (D) 101.1 dB

Common Data For Q. 41 and 42 :

In a PCM system, the bit error rate due to channel noise is 10^{-4}. Assume that the peak signal to noise ratio on the recovered analog signal needs to be at least 30 dB.

MCQ 6.1.41 What is the minimum number of quantizing steps that can be used to encode the analog signal into a PCM signal ?

(A) 32 (B) 16
(C) 64 (D) 8

MCQ 6.1.42 If the original analog signal had an absolute bandwidth of 2.7 kHz, what is the null bandwidth of the PCM signal ?

(A) 27 kHz (B) 13.5 kHz
(C) 10.4 kHz (D) 5.4 kHz

Common Data For Q. 43 to 45 :

A multi level digital communication system sends one of 16 possible levels over the channel every 0.8 ms.

MCQ 6.1.43 What is the number of bits corresponding to each level ?

(A) 16 bits/level
(B) 2 bits/level
(C) 4 bits/level
(D) 8 bits/level

MCQ 6.1.44 What is the baud rate ?

(A) 8 baud
(B) 8000 baud
(C) 1250 baud
(D) 1.25 baud

MCQ 6.1.45 What is the bit rate for the signal ?

(A) 5 k bits/sec (B) 1.25 k bits/sec
(C) 0.31 k bits/sec (D) 4 k bits/sec

Common Data For Q. 46 to 48 :

The information in an analog waveform is first encoded into binary PCM and then converted to a multilevel signal for transmission over the channel. The number of multilevels is eight. Assume that analog signal has a bandwidth of 2700 Hz and is to be reproduced at the receiver output with an accuracy of $\pm 1\%$ (full scale)

MCQ 6.1.46 What is the minimum bit rate of the PCM signal ?

(A) 900 bits/sec
(B) 32.4 k bits/sec
(C) 6 k bits/sec
(D) 1.11 k bits/sec

MCQ 6.1.47 The minimum baud rate of the multilevel signal is
(A) 97.2 k symbols/sec
(B) 32.4 k symbols/sec
(C) 4.05 k symbols/sec
(D) 10.8 k symbols/sec

MCQ 6.1.48 What is the minimum absolute channel bandwidth required for transmission of the PCM signal ?
(A) 10.8 kHz
(B) 5.4 kHz
(C) 12.8 kHz
(D) 21.6 kHz

Common Data For Q. 49 and 50 :

A binary waveform of 9600 bits/sec is converted into an octal (multilevel) waveform that is passed through a channel with a raised cosine roll of Nyquist filter characteristic. The channel has a conditioned phase response out to 2.4 kHz.

MCQ 6.1.49 What is the baud rate of the multilevel signal ?
(A) 3 k symbol/sec
(B) 28.8 k symbol/sec
(C) 3.2 k symbol/sec
(D) 9.6 k symbol/sec

MCQ 6.1.50 What is the roll off factor of the filter characteristic ?
(A) 2 (B) 0.5
(C) 3 (D) 1

Common Data For Q. 51 and 52 :

An analog signal is to be converted into PCM signal that is a binary polar NRZ line code. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine roll off type with roll off factor $\alpha = 0.5$.

MCQ 6.1.51 What is the maximum PCM bit rate that can be supported by this system without introducing ISI (intersymbol interference) ?
(A) 5.33 k bits/sec
(B) 8 k bits/sec
(C) 12 k bits/sec
(D) 16 k bits/sec

MCQ 6.1.52 The maximum bandwidth that can be permitted for the analog signal is
(A) 666 Hz
(B) 2.22 kHz
(C) 333 Hz
(D) 1.33 kHz

Common Data For Q. 53 and 54 :

Multilevel data with an equivalent bit rate of 2400 bits/sec is sent over a channel using a four level line code that has a rectangular pulse shape at the output of the transmitter. The overall transmission system (i.e., the transmitter, channel and receiver) has an $\alpha = 0.5$ raised cosine roll off Nyquist filter characteristic.

MCQ 6.1.53 What is the 6 dB bandwidth for this transmission system ?

(A) 2400 Hz (B) 1200 Hz

(C) 300 Hz (D) 600 Hz

MCQ 6.1.54 What is the absolute bandwidth for the system ?

(A) 900 Hz (B) 1800 Hz

(C) 450 Hz (D) 600 Hz

Common Data For Q. 55 to 57 :

A DM (Delta modulation) system is tested with a 10 kHz sinusoidal signal, 1 volt peak-to-peak, at the input. The signal is sampled at 10 times the Nyquist rate.

MCQ 6.1.55 What is the minimum step size required to prevent slope overload ?

(A) 0.157 volt (B) 0.314 volt

(C) 0.05 volt (D) 0.1 volt

MCQ 6.1.56 What is the power spectral density for the granular noise ?

(A) 1.23×10^{-7} V^2/Hz (B) 2.47×10^{-2} V^2/Hz

(C) 7.39×10^{-7} V^2/Hz (D) 2.06×10^{-8} V^2/Hz

MCQ 6.1.57 If the receiver input is band limited to 200 kHz, what is the average signal quantizing noise power ratio ?

(A) 11.8 dB (B) 1.18 dB

(C) 14.8 dB (D) 1.48 dB

MCQ 6.1.58 Five messages bandlimited to W, W, $2W$, $4W$ and $4W$ Hz, respectively are to be time division multiplexed. What is the minimum transmission bandwidth required for this TDM signal ?

(A) $24W$ (B) W

(C) $4W$ (D) $12W$

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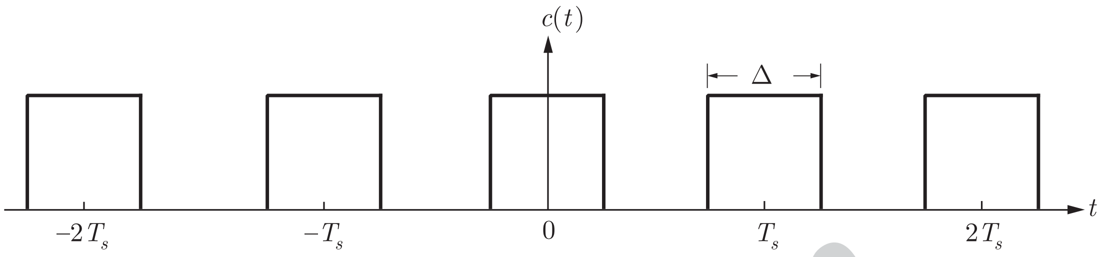
EXERCISE 6.2

QUES 6.2.1 Consider a set of 10 signals $x_i(t)$, $i = 1, 2, 3, ...10$. Each signal is band limited to 1 kHz. All 10 signals are to be time-division multiplexed after each is multiplied by a carrier $c(t)$ shown in figure.



If the period T_s of $c(t)$ is chosen that have the maximum allowable value, the largest value of $\triangle$ would be _ _ _ _ _ _ _ μs.

QUES 6.2.2 A compact disc recording system samples a signal with a 16-bit analog-to-digital convertor at 44.1 kHz. The CD can record an hour worth of music. The approximate capacity of CD is _ _ _ _ _ _ _ Mbytes.

QUES 6.2.3 An analog signal is sampled at 36 kHz and quantized into 256 levels. The time duration of a bit of the binary coded signal is _ _ _ _ _ _ _ μs.

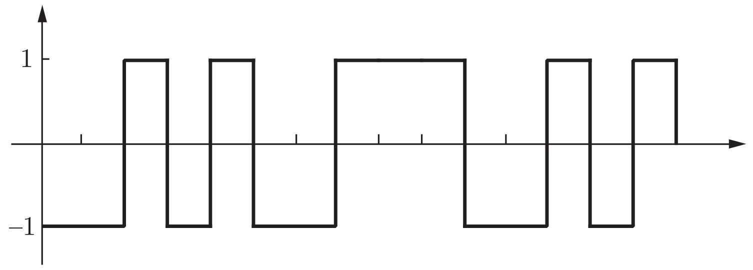
QUES 6.2.4 An analog signal is quantized and transmitted using a PCM system. The tolerable error in sample amplitude is 0.5% of the peak-to-peak full scale value. The minimum binary digits required to encode a sample is _ _ _ _ _ _ _

QUES 6.2.5 A Television signal is sampled at a rate of 20% above the Nyquist rate. The signal has a bandwidth of 6 MHz. The samples are quantized into 1024 levels. The minimum bandwidth required to transmit this signal would be _ _ _ _ _ _ _ Mbps.

QUES 6.2.6 A CD record audio signals digitally using PCM. The audio signal bandwidth is 15 kHz. The Nyquist samples are quantized into 32678 levels and then binary coded. What is the minimum bit rate (in kbps) required to encode the audio signal ?

QUES 6.2.7 The American Standard Code for information interchange has 128 characters, which are binary coded. If a certain computer generates 1,000,000 character per second, the minimum bandwidth required to transmit this signal will be _ _ _ _ _ _ _ Mbps.

QUES 6.2.8 Figure given below shows a PCM signal in which amplitude level of + 1 volt and -1 volt are used to represent binary symbol 1 and 0 respectively. The code word used consists of three bits. The sampled version of analog signal from which this PCM signal is derived is _______



QUES 6.2.9 A PCM system uses a uniform quantizer followed by a 8-bit encoder. The bit rate of the system is equal to 10^8 bits/s. The maximum message bandwidth for which the system operates satisfactorily is _______ MHz.

QUES 6.2.10 Twenty-four voice signals are sampled uniformly at a rate of 8 kHz and then time-division multiplexed. The sampling process uses flat-top samples with 1 μs duration. The multiplexing operation includes provision for synchronization by adding an extra pulse of 1 μs duration. What is the spacing (in μs) between successive pulses of the multiplexed signal ?

QUES 6.2.11 A linear delta modulator is designed to operate on speech signals limited to 3.4 kHz. The sampling rate is 10 times the Nyquist rate of the speech signal. The step size δ is 100 mV. The modulator is tested with a 1kHz sinusoidal test signal. The maximum amplitude of the test signal required to avoid slope overload is _______ mV.

QUES 6.2.12 The output signal-to-quantization-noise ratio of a 10-bit PCM was found to be 30 dB. The desired SNR is 42 dB. It can be increased by increasing the number of quantization level. In this way the percentage increase in the transmission bandwidth would be _______ %. (Assume $\log_2 10 = 0.3$)

QUES 6.2.13 For a PCM signal the compression parameter $\mu = 100$ and the minimum signal to quantization-noise ratio is 50 dB. The number of bits per sample would be _______

QUES 6.2.14 A sinusoid massage signal $m(t)$ is transmitted by binary PCM without compression. If the signal to-quantization-noise ratio is required to be at least 48 dB, the minimum number of bits per sample will be _______

QUES 6.2.15 A speech signal has a total duration of 20 sec. It is sampled at the rate of 8 kHz and then PCM encoded. The signal-to-quantization noise ratio is required to be 40 dB. The minimum storage capacity needed to accommodate this signal is _______ kbytes.

QUES 6.2.16 A sinusoidal signal with peak-to-peak amplitude of 1.536 V is quantized into 128 levels using a mid-rise uniform quantizer. The quantization-noise power is _______ $\times 10^{-6}$ V^2.

QUES 6.2.17 The Nyquist sampling interval, for the signal $\text{sinc}(700t) + \text{sinc}(500t)$ is _______ second.

QUES 6.2.18 In a PCM system, if the code word length is increased form 6 to 8 bits, then by what factor signal to quantization noise ratio improves ?

QUES 6.2.19 Four signals $g_1(t), g_2(t), g_3(t)$ and $g_4(t)$ are to be time division multiplexed and transmitted. $g_1(t)$ and $g_4(t)$ have a bandwidth of 4 kHz, and the remaining two signals have bandwidth of 8 kHz. Each sample requires 8 bits for encoding. What is the minimum transmission bit rate (kbps) of the system ?

QUES 6.2.20 Three analog signals, having bandwidths 1200 Hz, 600 Hz and 600 Hz, are sampled at their respective Nyquist rates, encoded with 12 bit words, and time division multiplexed. What is the bit rate (in kbps) for the multiplexed signal ?

QUES 6.2.21 Four signals each band limited to 5 kHz are sampled at twice the Nyquist rate. The resulting PAM samples are transmitted over a single channel after time division multiplexing. The theoretical minimum transmission bandwidth of the channel should be equal to _______ kHz.

QUES 6.2.22 Four independent messages have bandwidths of 100 Hz, 100 Hz, 200 Hz and 400 Hz respectively. Each is sampled at the Nyquist rate, time division multiplexed and transmitted. The transmitted sample rate in Hz, is given by _______

QUES 6.2.23 The Nyquist sampling rate for the signal $g(t) = 10\cos(50\pi t)\cos^2(150\pi t)$ where 't' is in seconds, is _______ samples per second.

QUES 6.2.24 A TDM link has 20 signal channels and each channel is sampled 8000 times/sec. Each sample is represented by seven binary bits and contains an additional bit for synchronization. The total bit rate for the TDM link is _______ kbps.

QUES 6.2.25 The Nyquist sampling interval for the signal $s(t) = \text{sinc}(350t) + \text{sinc}(250t)$ is _______ second.

QUES 6.2.26 In a CD player, the sampling rate is 44.1 kHz and the samples are quantized using a 16-bit/sample quantizer. The resulting number of Mega-bits for a piece of music with a duration of 50 minutes is _______

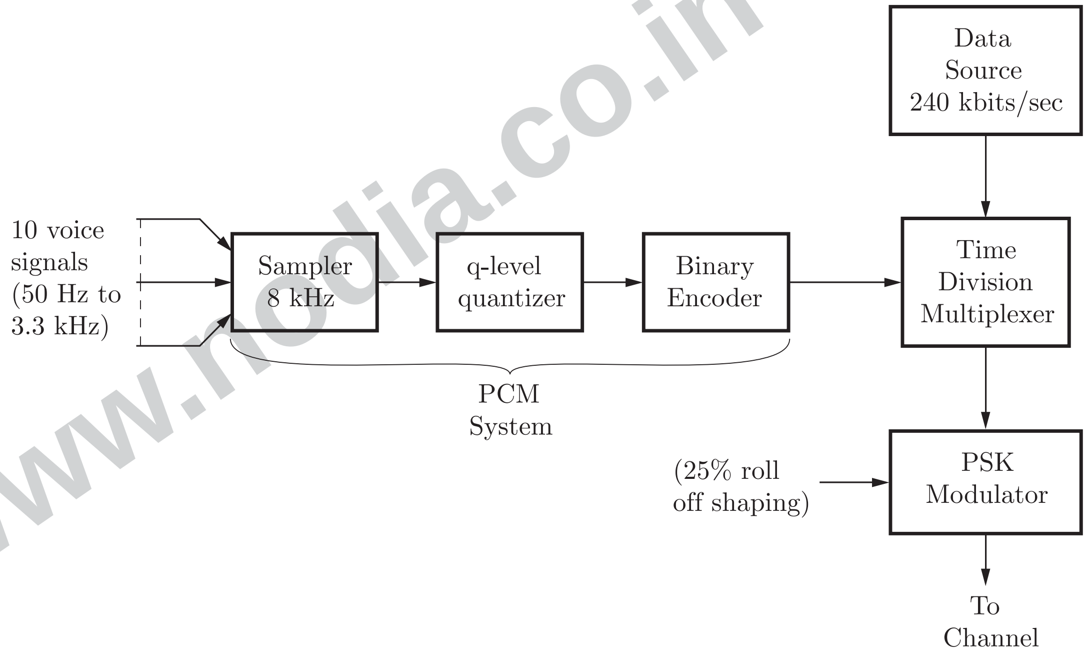
QUES 6.2.27 Four voice signals, each limited to 4 kHz and sampled at Nyquist rate are converted into binary PCM signal using 256 quantization levels. The bit transmission rate for the time-division multiplexed signal will be _______ kbps.

QUES 6.2.28 Analog data having highest harmonic at 30 kHz generated by a sensor has been digitized using 6 level PCM. What will be the rate (in kbps) of digital signal generated ?

QUES 6.2.29 In a PCM system, the number of quantization levels is 16 and the maximum signal frequency is 4 kHz; the bit transmission rate is _______ kbps.

QUES 6.2.30 A speech signal occupying the bandwidth of 300 Hz to 3 kHz is converted into PCM format for use in digital communication. If the sampling frequency is 8 kHz and each sample is quantized into 256 levels, then the output bit rate will be _______ kbps.

QUES 6.2.31 A PCM system is time division multiplexed with a data source, as shown in figure below. The output of the time division multiplexer is fed into a PSK modulator after which the signal is transmitted over a channel of bandwidth $B_T = 1\,\text{MHz}$ with center frequency at 100 MHz. What is the maximum number of quantization levels required in this system ?



QUES 6.2.32 A time division multiplexing system using PCM is used to multiplex telephone conversations over a single communication channel. It is found that a minima interval of $1\,\mu\text{s}$ must be allowed for reliable identification of bits at the receiving end. If the allowable voice bandwidth for telephone line transmission is 3 kHz and the quantization level is given as 16, what is the approximate number of voice signals that can be multiplexed ?

Common Data For Q. 33 and 34 :

Consider a PCM multiplexing system using a 256 level signal quantizer for the transmission of three signals m_1, m_2 and m_3, bandlimited to 5 kHz, 10 kHz, and 5 kHz, respectively. Assume that each signal is sampled at Nyquist rate and 8 bits transmitted simultaneously.

QUES 6.2.33 What is the maximum bit duration (in μs) for the transmitted signal ?

QUES 6.2.34 The channel bandwidth required to pass the PCM signal is _______ kHz.

QUES 6.2.35 An information signal to be transmitted digitally is a rectangular wave with a period of 71.4 μs. It has been determined that the wave will be adequately passed if the bandwidth includes the fourth harmonic. The minimum sampling frequency required for the signal is _______ kHz.

QUES 6.2.36 The voltage range of an A/D converter that uses 14 bit numbers is -6 to $+6$ V. What will be the resolution (in μV) of digitization ?

QUES 6.2.37 The input voltage of a compander with a maximum voltage range of 1 volt and $\mu = 255$ is 0.25 volt. What will be the voltage gain of compander ?

QUES 6.2.38 A multilevel digital communication system is to operate at a data rate of 9600 bits/sec. If 4 bit words are encoded into each level for transmission over the channel, what is the minimum required bandwidth (in Hz) for the channel ?

QUES 6.2.39 An analog signal is to be converted into a PCM signal that is multilevel polar NRZ line code with four number of levels. The signal is transmitted over a channel that is absolutely bandlimited to 4 kHz. Assume that the PCM quantizer has 16 steps and that the overall equivalent system transfer function is of the raised cosine roll off type with roll off factor $\alpha = 0.5$. What is the maximum bandwidth (in kHz) that can be permitted for the analog signal ?

QUES 6.2.40 Assume that a PCM type system is to be designed such that an audio signal can be delivered at the receiver output. The audio signal is to have a bandwidth of 3400 Hz and an SNR of at least 40 dB. What is the bit rate requirement (in kbps) for a design that uses $\mu = 255$ companded PCM signaling ?

QUES 6.2.41 A continuous data signal is quantized and transmitted using a PCM signal. If each data sample at the receiving end of the system must be known to within $\pm 0.5\%$ of the peak-to-peak full scale value, how many binary symbols must each transmitted digital word contain ?

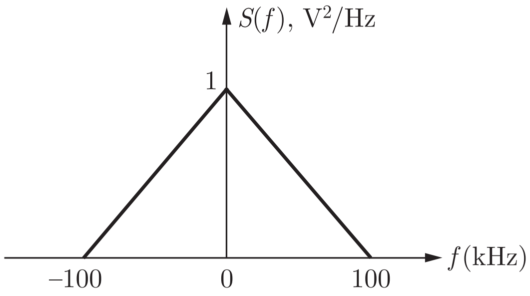
QUES 6.2.42 A delta modulator has the message signal,

$$m(t) = 4\sin 2\pi(10)t + 5\sin 2\pi(20)t$$

If the step size is $\delta = 0.05\pi$, what is the minimum sampling frequency (in Hz) required to prevent slope overload ?

QUES 6.2.43 A message signal has the following frequency components :
First component : A signal tone sine wave of 500 Hz.
Second component : A sound of frequency components with lowest value of 750 Hz and highest value of 1800 Hz.
What should be the minimum sampling frequency (in Hz) to sense the information present in this signal according to the sampling theorem ?

QUES 6.2.44 Consider a signal with power spectral density shown in figure below.



If the signal is sampled at 90% of its Nyquist rate and reconstruction filter has an ideal rectangular amplitude response, what will be signal to distortion ratio in decibel ?

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EXERCISE 6.3

MCQ 6.3.1 A signal that is mistakenly sampled when the sampling frequency is less than twice the input frequency is a/an
(A) harmonic
(B) pseudo signal
(C) alias
(D) false wave

MCQ 6.3.2 The smallest increment of voltage that the D/A converter produces over its output range is called the
(A) step
(B) resolution
(C) pixel
(D) bit

MCQ 6.3.3 A circuit that converts an instantaneous value of analog voltage into a binary number is the
(A) multiplexer
(B) half-adder
(C) D/A converter
(D) A/D converter

MCQ 6.3.4 In what type of multiplexing does each signal occupy the entire bandwidth of the channel?
(A) frequency-division multiplexing
(B) time-division multiplexing
(C) pulse-width multiplexing
(D) phase-shift multiplexing

MCQ 6.3.5 Which of the following is a very popular form of multiplexing where multiple channels of digital data are transmitted in serial form?
(A) frequency-division multiplexing
(B) phase-shift multiplexing
(C) pulse-amplitude modulated multiplexing
(D) pulse-code modulated multiplexing

MCQ 6.3.6 Which of the following is not a common type of media used in data communication?
(A) wire cable (B) radio
(C) wave guide (D) coaxial cable

MCQ 6.3.7 The main advantage of TDM over FDM is that it
(A) needs less power (B) needs less bandwidth

(C) needs simple circuitry
(D) gives better S/N ratio

MCQ 6.3.8 The PWM needs
(A) more power than PPM
(B) more samples per second than PPM
(C) more bandwidth than PPM
(D) none of the above

MCQ 6.3.9 The PAM signal can be detected by
(A) bandpass filter
(B) bandstop filter
(C) high pass filter
(D) low pass filter

MCQ 6.3.10 Flat-top sampling leads to
(A) an aperture effect
(B) aliasing
(C) loss of signal
(D) none

MCQ 6.3.11 In PCM, the quantization noise depends on
(A) sampling rate
(B) number of quantization levels
(C) signal power
(D) none of the above

MCQ 6.3.12 Which of the following modulation is digital in nature
(A) PAM
(B) PPM
(C) DM
(D) none of the above

MCQ 6.3.13 Which of the following modulation is analog in nature
(A) PCM
(B) DPCM
(C) DM
(D) none of these

MCQ 6.3.14 Quantization noise occurs in
(A) PAM
(B) PWM
(C) DM
(D) none of the above

MCQ 6.3.15 Companding is used in PCM to
(A) reduce bandwidth
(B) reduce power
(C) increase S/N ratio
(D) get almost uniform S/N ratio

MCQ 6.3.16 The main advantage of PCM is
(A) less bandwidth
(B) less power
(C) better performance in presence of noise
(D) possibility of multiplexing

MCQ 6.3.17 The main disadvantage of PCM is
(A) large bandwidth
(B) large power
(C) complex circuitry
(D) quantization noise

MCQ 6.3.18 The main advantage of DM over PCM is
(A) less bandwidth
(B) less power
(C) better S/N ratio
(D) simple circuitry

MCQ 6.3.19 In pulse analog modulation, with respect to message signal, the modulation is achieved by varying
(A) pulse amplitude
(B) pulse width
(C) pulse position
(D) all the pulse parameters

MCQ 6.3.20 Pulse amplitude modulation involves
(A) varying amplitude of message signal according to amplitude of pulse train
(B) performing amplitude modulation and then multiplying the result with pulse train
(C) varying amplitude of pulse train according to instantaneous variations of message signal
(D) performing multiplcation of pulse train with message and then subjecting the result to amplitude modulation.

MCQ 6.3.21 Pulse width modulation involves
(A) varying duration of message signal according to width of pulse train
(B) varying width of pulses in the pulse train according to instantaneous variations of message signal
(C) performing duration modification of message signal and then multiplying the result width pulse train
(D) performing width modification of pulse train with message and then subjecting the result to width modulation

MCQ 6.3.22 Pulse position modulation involves
(A) varying position of message signal components according to the position of pulses in the pulse train
(B) varying position of pulses in the pulse train according to the instantaneous variations in the message signal
(C) varying position of pulses in the pulse train according to the message components position
(D) performing position modification of pulse train with message and then subjecting the result to position modification

MCQ 6.3.23 Pulse code modulation involves
(A) PAM followed by quantization
(B) Direct encoding using binary words
(C) PAM followed by quantization and encoding using binary words
(D) PAM followed by encoding using binary words

MCQ 6.3.24 Delta modulation involves
(A) PAM followed by encoding using one-bit binary words
(B) PAM followed by quantization and encoding using one-bit binary words
(C) PAM followed by one-bit quantization
(D) direct encoding using one-bit quantization

MCQ 6.3.25 Sampling is the process which convert :
(A) discrete signal to continuous signal
(B) analog signal to digital signal

(C) continuous signal to discrete signal

(D) digital signal to analog signal

MCQ 6.3.26 Which is correct for Nyquist rate

if f_s = frequency of sampling

f_m = highest frequency of band limited signal

(A) $f_s = f_m$

(B) $f_s > 2f_m$

(C) $f_m > 2f_s$

(D) $f_s < 2f_m$

MCQ 6.3.27 Amplitude of pulse train varies according to instantaneous value of modulating signal is called :

(A) PLM

(B) PDM

(C) PAM

(D) PWM

MCQ 6.3.28 Quantization noise occurs in :

(A) TDM

(B) PCM

(C) FDM

(D) None of the above

MCQ 6.3.29 Several signals can be time division multiplexed (TDM) because

(A) these signals cannot be sampled

(B) be ideally sampled

(C) be sampled with any arbitrary sample shape

(D) none of the above

MCQ 6.3.30 Aliasing occurs due to

(A) over sampling

(B) under sampling

(C) attenuation or amplification of signals

(D) none of the above

MCQ 6.3.31 A PAM signal is recovered by using

(A) low pass filter

(B) high pass filter

(C) band pass filter

(D) none of the above

MCQ 6.3.32 Pulse modulation is used in

(A) Radio navigation

(B) Automatic landing system

(C) Data Communications

(D) All of a above

MCQ 6.3.33 PAM signal can be demodulated by using

(A) a low pass filter

(B) a band pass filter

(C) a high pass filter

(D) None of the above

MCQ 6.3.34 In a pulse position modulation system, the transmitted pulse have

(A) Constant amplitudes but varying widths

(B) Constant amplitudes constant widths

(C) Constant width by varying amplitudes

(D) None of the above

MCQ 6.3.35 A "PWM" signal can be generated by

(A) a monostable multivibrator

(B) an astable multivibrator

(C) Integrating the PPM signal

(D) Differenting the PPM signal

MCQ 6.3.36 Time division multiplexing

(A) Can be used only with PCM

(B) Combines five groups into a super-group

(C) Stacks 24 channels in adjacent frequency slots

(D) Interleaves pulse belonging to different transmission

MCQ 6.3.37 If the transmission bandwidth is W and available channel bandwidth is W_{Channel}, what should be the condition that will allow fruitful reception?

(A) $W = W_{\text{channel}}$ (B) $W < W_{\text{channel}}$

(C) $W > W_{\text{channel}}$ (D) All of the above

MCQ 6.3.38 In pulse modulation systems, the number of samples required to ensure no loss of information is given by

(A) Fourier transform (B) Nyquist theorem

(C) Parseval's theorem (D) Shannon's Theorem

MCQ 6.3.39 A PCM receiver can see

(A) Quantization nosie (B) Channel noise

(C) Interference noise (D) All of the above

MCQ 6.3.40 In PCM for q quantizing levels, the number of pulses p in a code group is given by

(A) $\log_{10} q$ (B) $\log_2 q$

(C) $\ln q$ (D) $2\log_2 q$

MCQ 6.3.41 The principal merit of PCM system is its

(A) Lower bandwidth

(B) Lower noise

(C) Lower power requirement

(D) Lower cost

MCQ 6.3.42 A compander is used in communication systems to

(A) Compress the bandwidth

(B) Improve the frequency response

(C) Reduce the channel noise

(D) Improve singal-to-noise ratio

MCQ 6.3.43 Which of the following systmes is digital?

(A) PPM (B) PWM

(C) PCM (D) PFM

MCQ 6.3.44 Which of the following is unlikely to happen when the quantizing noise is decreased in PCM?

(A) Increase in the bandwidth

(B) Increase in the number of standard levels

(C) Increase in the channel noise

(D) All of the above

MCQ 6.3.45 Which of the following systems is not digital?

(A) Differential PCM (B) DM

(C) ADPCM (D) PAM

MCQ 6.3.46 In which of the following methods of source coding, maximum compression is achieved?

(A) A-law PCM (B) μ-law PCM

(C) DM (D) ADPCM

MCQ 6.3.47 A sine wave carrier cannot be modified by the intelligence signal through which of the following?

(A) amplitude modulation

(B) pulse modulation

(C) frequency modulation

(D) phase modulation

****

SOLUTIONS 6.1

SOL 6.1.1 Option (B) is correct.

From the Fourier spectra of signal $x(t)$, we observe that the bandwidth of $x(t)$ is

$$\omega_x = 2\pi \times 10^5$$

or, $$2\pi f_x = 2\pi \times 10^5$$

or, $$f_x = 10^5 \text{ Hz}$$

So, the Nyquist sampling rate for $x(t)$ is given by

$$\begin{aligned} f_s &= 2f_x \\ &= 2 \times 10^5 \text{ Hz} \\ &= 200 \text{ kHz} \end{aligned}$$

SOL 6.1.2 Option (D) is correct.

From the Fourier spectrum of signal $y(t)$, we obtain the bandwidth of $y(t)$ as

$$\omega_y = 3\pi \times 10^5$$

$$2\pi f_y = 3\pi \times 10^5$$

or, $$\begin{aligned} f_y &= 1.5 \times 10^5 \text{ Hz} \\ &= 150 \text{ kHz} \end{aligned}$$

So, the Nyquist sampling rate for $y(t)$ is

$$\begin{aligned} f_s &= 2f_y \\ &= 2 \times 150 \\ &= 300 \text{ kHz} \end{aligned}$$

SOL 6.1.3 Option (D) is correct.

Again, we have the bandwidth the signal $x^2(t)$ as

$$\begin{aligned} f_{x^2} &= 2f_x \\ &= 2 \times 10^5 \text{ Hz} \end{aligned}$$

So, the Nyquist sampling rate of $x^2(t)$ is given by

$$\begin{aligned} f_s &= 2f_{x^2} \\ &= 2 \times (2 \times 10^5) \\ &= 4 \times 10^5 \text{ Hz} \\ &= 400 \text{ kHz} \end{aligned}$$

SOL 6.1.4 Option (C) is correct.

The bandwidth of signal $y^3(t)$ will be thrice that of signal $y(t)$. So, we have the bandwidth of $y^3(t)$ as

$$\begin{aligned} f_{y^3} &= 3f_y \\ &= 3 \times 1.5 \times 10^5 \\ &= 4.5 \times 10^5 \end{aligned}$$

So, the Nyquist sampling rate of $y^3(t)$ is given by

$$f_s = 2f_{y^3} = 2 \times (4.5 \times 10^5)$$

$$= 9 \times 10^5 \text{ Hz}$$
$$= 900 \text{ kHz}$$

SOL 6.1.5 Option (B) is correct.

The bandwidth of signal $x(t)y(t)$ is obtained as the sum of the bandwidth of $x(t)$ and $y(t)$. i.e.

$$f_{xy} = f_x + f_y$$
$$= 10^5 + 1.5 \times 10^5$$
$$= 2.5 \times 10^5 \text{ Hz}$$

So, the Nyquist sampling rate of $x(t)y(t)$ is given by

$$f_s = 2f_{xy}$$
$$= 2 \times (2.5 \times 10^5)$$
$$= 5 \times 10^5 \text{ Hz}$$
$$= 500 \text{ kHz}$$

SOL 6.1.6 Option (B) is correct.

From the shown pulse waveform of carrier signal $c(t)$, we have the sampling frequency

$$f_s = \frac{1}{T_s} = \frac{1}{10^{-3}} = 10^3 \text{ Hz.}$$

This sampling frequency must satisfy

The Nyquist critesion for sampling the signal $x(t)$ i.e.,

$$f_s > 2f_x$$

or $$2\pi f_s > 2\pi(2f_x)$$

$$2\pi \times 1000 > 2\omega$$

or $$\omega < 1000\pi$$

So, the signal $X(j\omega)$ must be defined for the region $\omega < 1000\pi$. Therefore, we have $X(j\omega) = 0$ for $\omega > 1000\pi$

SOL 6.1.7 Option (C) is correct.

From the shown rectangular pulse train, we have the time period of $c(t)$,

$$T_s = 10^{-3} \text{ sec}$$

ON Time duration of pulse train is

$$\triangle = 0.25 \text{ ms} = 0.25 \times 10^{-3} \text{ sec}$$

So, the duty cycle of pulse-train is given by

$$D = \frac{\triangle}{T_s} = \frac{0.25 \times 10^{-3}}{10^{-3}} = 0.25$$

Now, the gain of LPF is A. To recover the signal $x(t)$, the overall gain of the system must be 1. So, we have

$$D \times A = 1$$
$$\frac{\triangle}{T_s} A = 1$$
$$A = \frac{T_s}{\triangle} = \frac{1}{0.25} = 4$$

SOL 6.1.8 Option (B) is correct.

Given the bandwidth of each telemetry signal, $f_m = 2$ kHz.

Let the peak signal amplitude be m_p. So, we have the maximum tolerable error

$$\text{tolerable error} = 0.2\% \text{ of peak signal amplitude}$$

$$= \frac{0.2}{100} \times m_p$$

$$= \frac{m_p}{500} \qquad ...(1)$$

Now, the Nyquist sampling rate for each signal is

$$f_N = 2f_m = 2 \times 2 \times 1000$$
$$= 4000 \text{ Hz}$$

Since, the sampling must be 20% above the Nyquist rate so, we have the actual sampling rate as

$$f_s = 1.2 \times f_N = 1.2 \times 4000$$
$$= 4800 \text{ Hz}$$

Again, let the number of quantization level of the sampled signal be q. So, we have the maximum quantization error in the sampled signal as

$$\text{quantization error} = \frac{\frac{2m_p}{q}}{2} = \frac{m_p}{q} \qquad ...(2)$$

This error must be less than the maximum tolerable error. So, from equations (1) and (2), we have

$$\frac{m_p}{q} \le \frac{m_p}{500}$$

or, $\quad q \ge 500$

or $\quad 2^n \ge 500$

where n is the number of bits used to encode the quantized signal. For satisfying the above condition, the minimum number of bits is $n = 9$ bits. Therefore, the bit rate of the sampled signal is given by,

$$R_b = nf_s$$
$$= 9 \times 4800$$
$$= 4.32 \times 10^4 \text{ bits/sec}$$

since, ten signals are multiplexed, and framing and synchronising requires an additional 1% extra bits. Therefore, the total data rate is given by

$$\text{Data rate} = 1.01 \times [10 \times R_b]$$
$$= 1.01 \times 10 \times (4.32 \times 10^4)$$
$$= 4.3632 \times 10^5 \text{ bits/sec}$$
$$= 436.32 \text{ kbits/sec}$$

SOL 6.1.9 Option (A) is correct.

We have just obtained the minimum possible data rate,

$$\text{Data rate} = 436.32 \text{ kbits/sec}$$

So, the minimum transmission bandwidth of the multiplexed signal is given by

$$B_T = \frac{\text{Data rate}}{2}$$
$$= \frac{436.32}{2}$$
$$= 218.16 \text{ kHz}$$

NOTE :

As we can transmit error free at most two peices of information per second per Hz bandwidth, here we have dividided the data rate by 2 bits (information) to get the minimum transmission bandwidth.

SOL 6.1.10 Option (D) is correct.

Given the transmission bandwidth capacity of binary channel,

$$B_{ch} = 36 \text{ kbits/sec}$$

Bandwidth of message signal,

$$f_m = 3.2 \text{ kHz}$$

So, the sampling frequency of signal must be

$$f_s \geq 2f_m$$

or, $$f_s \geq 2 \times (3.2) = 6.4 \text{ kHz} \qquad ...(1)$$

Now, let the quantization level be q. So, we have $q = 2^n$, where n is the number of bits required to encode the sample. So, we have the bit rate for the signal,

$$R_b = nf_s$$

Therefore, the maximum transmission bandwidth for the signal is given by

$$B_T = R_b = nf_s$$

This transmision bandwidth must be less than the bandwidth capacity of binary channel i.e.

$$R_b \leq B_{ch}$$

or, $$nf_s \leq 36 \qquad ...(2)$$

Now, checking the options for inequality (1), we have the sampling frequency $f_s = 7.2$ kHz.

Substituting it in equation (2), we get

$$7.2n \leq 36$$

or $$n \leq 5$$

or $$2^n \leq 2^5$$

or $$q \leq 32$$

Thus, the appropriate values of quantizing level and sampling frequency are $q = 32$ and $f_s = 7.2$ kHz

SOL 6.1.11 Option (D) is correct.

Given the bandwidth of message signal for which delta modulator is designed is

$$B = 3.5 \text{ kHz}$$

the amplitude of test signal, $A_m = 1$ volt

frequency of test signal, $f_m = 800$ Hz

sampling frequency of the system, $f_s = 64$ kHz

So, we have time duration of a sample as

$$T_s = \frac{1}{64 \times 10^3} = 1.56 \times 10^{-5} \text{ sec}$$

Let the step size of the delta modulated signal be δ. So, the condition to avoid slope overload is

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or $$\frac{\delta}{1.56 \times 10^{-5}} \geq A_m(2\pi f_m)$$

or, $$\delta \geq (1.56 \times 10^{-5}) \times 1 \times (2\pi \times 800)$$

or, $$\delta \geq 7.84 \times 10^{-2} \text{ volt}$$

Thus, the minimum value of step size to avoid slope overload is

$$\delta = 78.5 \text{ mV}$$

SOL 6.1.12 Option (D) is correct.

Again, we have the analog signal band for which delta modulator is designed as $B = 3.5\,\text{kHz}$.

Sampling frequency of the system, $f_s = 64\,\text{kHz}$.

The step size, we have just obtained as $\delta = 78.5\,\text{mV} = 78.5 \times 10^{-3}$

So, the granular noise power in the analog signal band is given by

$$N = \frac{\delta^2 B}{3f_s}$$
$$= \frac{(78.5 \times 10^{-3})^2 \times (3.5 \times 10^3)}{3 \times (64 \times 10^3)}$$
$$= 1.123 \times 10^{-4} \text{ watt}$$

SOL 6.1.13 Option (C) is correct.

We have just obtained the granular noise power as

$$N = 1.12 \times 10^{-4} \text{ watt}$$

Also, we have the amplitude of message signal (sinusoidal test signal)

$$A_m = 1 \text{ volt}$$

So, the signal power is given by

$$S = \frac{A_m^2}{2} = \frac{1}{2} = 0.5 \text{ watt}$$

Therefore, SNR is given by

$$\text{SNR} = \frac{S}{N} = \frac{0.5}{1.12 \times 10^{-4}} = 4.46 \times 10^3$$

SOL 6.1.14 Option (B) is correct.

Given, the number of quantization level $q = 256$

For the μ-law quantizer, we have $\mu = 225$

So, according to μ-law companding, signal to quantization noise ratio is given by

$$\frac{S_o}{N_o} = \frac{3q^2}{[\ln(\mu+1)]^2}$$
$$= \frac{3 \times (256)^2}{[\ln(255+1)]^2}$$
$$= 6.39 \times 10^3$$

In decibel, the SNR is

$$\left(\frac{S_o}{N_o}\right)_{\text{db}} = 10\log_{10}(6.39 \times 10^3)$$
$$= 38.06 \text{ dB}$$

SOL 6.1.15 Option (C) is correct.

Given, the bandwidth of message signal, $W_m = 1$ MHz

So, the Nyquist rate for the signal is $f_N = 2W_m = 2$ MHz

Since, the signal to be sampled 50% higher than the Nyquist rate, we have

$$f_{s1} = 1.5 \times f_N = 3 \text{ MHz}$$

Also, we have the number of quantization level for modulating the signal,

$$q_1 = 256$$

or $$2^{n_1} = 256$$

or $$n_1 = 8 \text{ bits}$$

This is the number of bits required to sample the signal in first case. So, the transmission bandwidth for this case is

$$B_{T1} = n_1 f_{s1} = 8 \times 3 = 24 \text{ Mbits/sec}$$

Now signal to be sampled at a rate 20% above the Nyquist rate (adequate rate). So ,we have the sampling frequency for the second case as

$$f_{s2} = 1.2 f_N = 2.4 \text{ MHz}$$

Again, the transmission bandwidth must be same as in previous case. So, we have

$$B_{T2} = B_{T1} = 24 \text{ M bits/sec.}$$

or, $$n_2 f_{s2} = 24$$

or, $$n_2 = \frac{24}{2.4} = 10 \text{ bits}$$

This is the number of bits required to encode the signal in second case. So, we have the number of quantization level in the second case as

$$q_2 = 2^{n_2} = 2^{10}$$

Thus, the SNR in this case would be

$$\frac{S_o}{N_o} = \frac{3q_2^2}{[\ln(1+\mu)]^2}$$

$$= \frac{3 \times (2^{10})^2}{[\ln(1+255)]^2} = \frac{3 \times 2^{20}}{[\ln(256)]^2}$$

$$= 1.023 \times 10^5$$

In decibel, the SNR is

$$\left(\frac{S_o}{N_o}\right)_{\text{dB}} = 10 \log_{10}(1.023 \times 10^5)$$

$$= 50.1 \text{ dB}$$

SOL 6.1.16 Option (B) is correct.

Given the step size of linear delta modulator, $\delta = 0.628$

The sampling frequency, $f_s = 40 \text{ kHz} = 40 \times 10^3 \text{ Hz}$.

The condition for slope overload in delta modulation is

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\delta f_s \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq (0.628) \times 40 \times 10^3 = 2.512 \times 10^4 \qquad \text{...(1)}$$

Since, the input to delta modulator is sine wave with frequency f_m and peak amplitude A_m. So, we have

$$\max\left|\frac{dm(t)}{dt}\right| = A_m(2\pi f_m) \qquad \text{...(2)}$$

Therefore, from equations (1) and (2), we have

$$A_m(2\pi f_m) \leq 2.512 \times 10^4$$

or, $$A_m f_m \leq \frac{2.512}{2\pi} \times 10^4 = 3.998 \times 10^3$$

This is the condition to avoid slope overload.

In the given options, (A), (C) and (D) satisfy this condition, where as option (B) does not satisfy the condition. Therefore, the slope overload will take place at

$$A_m = 1.5 \text{ V}, \ f_m = 4 \text{ kHz}$$

SOL 6.1.17 Option (B) is correct.

Given, the sampling frequency, $f_s = 8$ kHz

Number of bits required for quantization, $n = 8$ bits

So, the bit rate of the PCM signal is given by

$$\begin{aligned} R_b &= nf_s \\ &= 8 \times 8 \text{ kHz} \\ &= 64 \text{ kbits/sec} \end{aligned}$$

Also, the SNR for quantization of sinusoidal signal obtained as

$$\begin{aligned} \text{SNR}_q &= \frac{S_o}{N_o} = 3q^2 \frac{\overline{m^2(t)}}{m_p} \\ &= 3q^2 \frac{1/2}{1} = \frac{3(2^n)^2}{2} = \frac{3(2^8)^2}{2} \\ &= 9.83 \times 10^4 \end{aligned}$$

In decidel, we have

$$\text{SNR}_q = 10\log_{10}(9.83 \times 10^4) = 49.93 \text{ dB}$$

SOL 6.1.18 Option (B) is correct.

Given the bandwidth of message signal

$$W = 1 \text{ kHz} = 1000 \text{ Hz}$$

Sampling frequency

$$\begin{aligned} f_s &= 1800 \text{ sample/sec} \\ &= 1800 \text{ Hz} \end{aligned}$$

Since, from the Nyquist criterion, we know that the sampling rate must be greater than the Nyquist rate given as

$$f_s \geq f_N = 2f_m$$

So, for the sampling frequency $f_s = 1800$ Hz, the message signal recovered has the maximum frequency,

$$f_m \leq \frac{f_s}{2} = \frac{1800}{2} = 900 \text{ Hz}$$

Thus, for the applied message signal of bandwidth $W = 1 \text{ kHz} = 1000 \text{ Hz}$, the recovered message signals are of the frequency less or equal to 900 Hz. i.e. the output of the filter contains 800 Hz and 900 Hz components.

SOL 6.1.19 Option (D) is correct.

Given the message signal

$$x(t) = 100\cos(24\pi \times 10^3 t)$$

Sampling period, $T_s = 50\,\mu\text{sec}$

Cut off frequency of the low pass filter $f_c = 15$ kHz

So, we have the sampling frequency

$$f_s = \frac{1}{T_s} = \frac{1}{50 \times 10^{-6}} = 20 \text{ kHz}$$

Also, the frequency of the message signal

$$f_m = \frac{24\pi \times 10^3}{2\pi} = 12 \text{ kHz}$$

Now, for a message signal of frequency f_m and sampling frequency f_s, the output consists the frequency components as $nf_s \pm f_m$, where $n = 0, 1, 2...$

So, for the given signal, we have frequency components of sampled output as

$$\begin{aligned} f_m &= 12 \text{ kHz} \\ f_s + f_m &= 20 + 12 = 22 \text{ kHz} \\ f_s - f_m &= 20 - 12 = 8 \text{ kHz} \\ 2f_s + f_m &= 40 + 12 = 52 \text{ kHz} \\ 2f_s - f_m &= 40 - 12 = 28 \text{ kHz and so on} \end{aligned}$$

When these signal components are passed through low pass filter of cut-off frequency $f_c = 15$ kHz, we get the output with frequency components $f_m = 12$ kHz, $f_s - f_m = 8$ kHz

SOL 6.1.20 Option (C) is correct.

Given the message signal

$$x(t) = 5\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^3 + 7\left(\frac{\sin 2\pi 1000t}{\pi t}\right)^2$$

For this signal, we have the maximum frequency

$$f_m = 3 \times 1000 = 3000 \text{ Hz}$$

So, the minimum sampling frequency (Nyquist frequency) required to reconstruct the signal is

$$\begin{aligned} f_s &= 2f_m \\ &= 2 \times 3000 = 6000 \text{ Hz} \\ &= 6 \times 10^3 \text{ Hz} \end{aligned}$$

SOL 6.1.21 Option (B) is correct.

Given the message signal,

$$m(t) = 12s[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)]$$

Sampling frequency,

$$\begin{aligned} f_s &= 32 \text{ k samples/sec} \\ &= 32 \text{ kHz} \end{aligned}$$

Now, condition to avoid the slope overload is given as

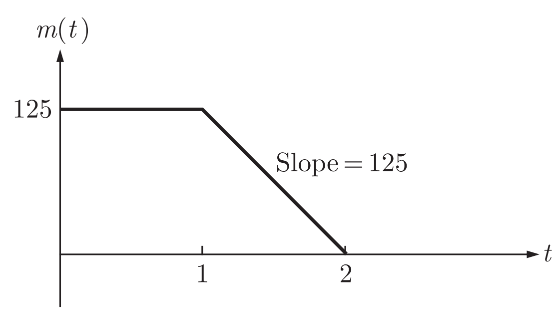
$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

where δ is the step size of the delta modulator and T_s is the sampling interval.

For the given message signal, we obtain

$$\begin{aligned} m(t) &= 125[u(t) - u(t-1)] + (250 - 125t)[u(t-1) - u(t-2)] \\ &= 125u(t) - 125(t-1)u(t-1) + 125(t-2)u(t-2) \end{aligned}$$

The waveform for this signal is shown below



So, from the waveform it is observed that

$$\max\left|\frac{dm(t)}{dt}\right| = 125$$

Therefore, the condition to avoid slope overload becomes

$$\frac{\delta}{T_s} \geq 125$$

or
$$\delta \geq \frac{125}{f_s} = \frac{125}{32 \times 10^3} = \frac{1}{256}$$

So, the minimum value of step size is

$$\delta = \frac{1}{256} = 2^{-8}$$

SOL 6.1.22 Option (D) is correct.

The signal to quantization noise ratio for a PCM system is given by

$$\text{SNR} = \frac{S_o}{N_o} = 3q^2 \frac{\overline{m^2(t)}}{m_p}$$

where q is the number of quantization levels, $\overline{m^2(t)}$ is the power in the message signal $m(t)$ and m_p is the peak amplitude of $m(t)$. Since, the number of bits in PCM system is increased from n to $n+1$, so we have the number of quantization levels in the two cases as

$$q_1 = 2^n$$
$$q_2 = 2^{n+1}$$

Therefore, The factor by which the new SNR increase is given by

$$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{q_2^2}{q_1^2}$$
$$= \frac{(2^{n+1})^2}{(2^n)^2} = 4 \times \frac{2^{2n}}{2^{2n}} = 4$$

SOL 6.1.23 Option (C) is correct.

The transmission bandwidth of a channel for PCM system is given by

$$B_T = R_b = nf_s \qquad ...(1)$$

where R_b is the bit rate, n is the number of bits per sample, and f_s is the sampling frequency. Also, the quantization level are defined as

$$q = 2^n$$

So, we have

$$n = \log_2 q$$

Substituting it in equation (1), we have

$$B_T = n\log_2 q$$

So, for change in the quantization level from $q_1 = 2$ to $q_2 = 8$, the change in bandwidth is obtained as

$$\frac{B_{T_1}}{B_{T_2}} = \frac{n\log_2 q_1}{n\log_2 q_2} = \frac{n\log_2 2}{n\log_2 8} \backslash$$
$$3BT_1 = B_{T_2}$$

i.e., the bandwidth requirement will be tripled.

SOL 6.1.24 Option (A) is correct.

Given, the highest frequency tone to be recorded,

$$f_m = 15800 \text{ Hz}$$

So, according Nyquists sampling criterion, the minimum sampling frequency required for the signal is

$$f_s = f_{\text{Nyquist}} = 2f_m$$
$$= 2 \times 15800$$
$$= 31600 \text{ Hz}$$
$$= 3.16 \times 10^4 \text{ Hz}$$

SOL 6.1.25 Option (B) is correct.

We have the minimum sampling frequency for the recording,

$$f_s = 3.16 \times 10^4 \text{ Hz}$$
$$= 3.16 \times 10^4 \text{ samples/ sec}$$

So, the number of sample required to store a three minutes performance is

$$\text{Number of sample} = f_s \times \text{Time duration}$$

$$= (3.16 \times 10^4)(3 \times 60)$$
$$= 5.688 \times 10^6 \text{ samples}$$

SOL 6.1.26 Option (C) is correct.

Given, the number of quantized levels,

$$q = 128 \text{ levels.}$$

Also, we have the number of samples required to store the three minutes performance

$$\text{no. of samples} = 5.688 \times 10^6 \text{ samples}$$

For 128 quantized levels, required number of bits is obtained as

$$2^n = 128$$
$$n = 7 \text{ bits}$$

Thus, 7 bits per sample are used to quantify the signal. Therefore, we have the number of bits required to store the three minutes performance as

$$\text{number of bits} = (\text{number of samples}) \times (\text{number of bits per sample})$$
$$= (5.688 \times 10^6) \times 7$$
$$= 3.98 \times 10^7 \text{ bits}$$

SOL 6.1.27 Option (B) is correct.

Given the rms voltage at sending end, $v_i = 1 \text{ volt rms}$

Length of the cable, $l = 1000 \text{ m}$

Attenuation in the cable, $\alpha = 1 \text{ dB/m}$

So, we have the total attenuation in the cable as

$$\alpha l = 1000 \text{ dB}$$

Thus, the rms voltage at receiving end is obtained as

$$20\log_{10}\left(\frac{v_o}{v_i}\right) = -1000 \text{ dB}$$
$$\log_{10}\frac{v_o}{v_i} = -50$$
$$v_o = v_i 10^{-50} = 1 \times 10^{-50} = 10^{-50} \text{ volt}$$

SOL 6.1.28 Option (B) is correct.

Given, the voltage gain of repeater

$$G = 100 \text{ V/V}$$

So, the gain in decide is obtained as

$$G = 20\log_{10}100$$
$$= 40 \text{ dB}$$

Since, the cable provides the attenuation of 1 dB/m. So, we need a repeater for every 40 m of cable. Thus, the total number of repeater required along the cable is

$$\text{no. of repeater} = \frac{\text{length of cable}}{40}$$
$$= \frac{1000}{40} = 25$$

SOL 6.1.29 Option (A) is correct.

Given, number of quantization levels, $q = 4$. Since, the random signal ranges between

$$-4 \le X(t) \le 4 \text{ volt}$$

So, we have the peak-to-peak amplitude of the signal as

$$2m_p = 4-(-4) = 8 \text{ volt}$$

Therefore, the step size of each level is

$$\delta = \frac{2m_p}{q} = \frac{8}{4} = 2 \text{ volt}$$

SOL 6.1.30 Option (D) is correct.

Given, the probability density of signal $X(t)$,

$$f_{X(t)}(x) = \begin{cases} ke^{-|x|}, & |X| \le 4 \text{ volt} \\ 0 \quad , & \text{otherwise} \end{cases}$$

From the property of probability density function, we know that

$$\int_{-\infty}^{\infty} f_{X(t)}(x)dx = 1$$

So, we solve the above equation for value of k as

$$\int_{-4}^{4} ke^{-|x|}dx = 1$$

or $$2k\int_0^4 e^{-x}dx = 1$$

or $$2k\left[\frac{e^{-x}}{-1}\right]_0^4 = 1$$

or $$2k[-e^{-4}+1] = 1$$

Thus, $$k = \frac{0.5}{1-e^{-4}} = 0.5093$$

SOL 6.1.31 Option (D) is correct.

We have the four quantization levels in the range $-4 \le X \le 4$ given by

$$x_1 = -3 \text{ volt}$$
$$x_2 = -1 \text{ volt}$$
$$x_3 = 1 \text{ volt}$$
$$x_4 = 3 \text{ volt}$$

So, for any value between -2 to -4, the quantized value is -3. So for any value x in the range $-2 < x < 4$, the quantization error is

$$e(X) = (-3-x)$$

Similar discussions can be made for other quantization levels. So, we get the variance of quantization error is

$$\begin{aligned}
\text{var}[e(X)] &= \int_{-4}^{-2}(-3-x)^2 ke^{-|x|}dx + \int_{-2}^{0}(-1-x)^2 ke^{|-x|}dx \\
&\qquad + \int_0^2 (1-x)^2 ke^{-|x|}dx + \int_2^4 (3-x)^2 ke^{-|x|}dx \\
&= 2k\int_0^2 (1-x)^2 e^{-x}dx + 2k\int_2^4 (3-x)^2 e^{-x}dx \\
&= 2k\left[(1-x)^2(-e^{-x}) - \{-2(1-x)\}e^{-x} + 2(-e^{-x})\right]_0^2 \\
&\qquad + 2k\left[(3-x)^2(-e^{-x}) - \{-2(3-x)\}e^{-x} + 2(-e^{-x})\right]_2^4 \\
&= 2k[e^{-x}(-1-x^2+2x+2-2x-2)]_0^2 \\
&\qquad + 2k[e^{-x}(-9-x^2+6x+6-2x-2)]_0^4 \\
&= 2k[e^{-x}(-1-x^2)]_0^2 + 2k[e^{-x}(-5-x^2+4x)]_2^4 \\
&= 2k[e^{-2}(-5)-(-1)] + 2k\begin{bmatrix} e^{-4}(-5-16+16) \\ \qquad -e^{-2}(-5-4+8) \end{bmatrix} \\
&= 2\times 0.5093[1-5e^{-2}+e^{-2}-5e^{-4}] \\
&= 0.3739
\end{aligned}$$

SOL 6.1.32 Option (A) is correct.

Given, the message signals

$$m_1(t) = \operatorname{sinc}(100t) = \frac{\sin(100\pi t)}{100\pi t}$$

$$m_2(t) = \operatorname{sinc}^2(100t) = \frac{\sin^2(100\pi t)}{(100\pi t)^2}$$

$$= \frac{1}{(100\pi t)^2}\left[\frac{1-\cos(200\pi t)}{2}\right]$$

So, we have the corresponding maximum frequency for these signals as

$$f_{m1} = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

$$f_{m2} = \frac{200\pi}{2\pi} = 100 \text{ Hz}$$

Therefore, the corresponding Nyquist rates are obtained as

$$f_{N_1} = 2f_{m1} = 2\times 50\,Hz = 100 \text{ Hz}$$

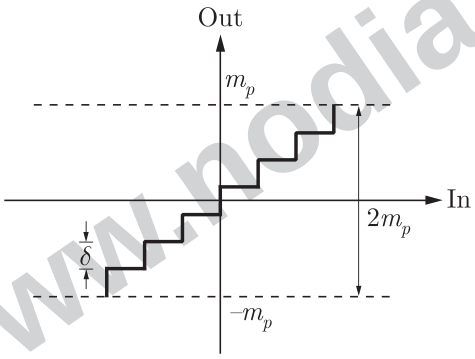
$$f_{N_2} = 2f_{m2} = 2\times 100 = 200 \text{ Hz}$$

SOL 6.1.33 Option (C) is correct.

Let the Binary PCM consists n bits per sample. So, we have the quantization level,

$$q = 2^n \text{ Levels}$$

A quantized signal with peak-to-peak value $2m_p$ is shown in figure below.



From the figure, we have the step size

$$\delta = \frac{2m_p}{q}$$

So, the maximum quantization error is given by

$$|\text{error}| = \frac{\delta}{2} = \frac{m_p}{q}$$

Since, the quantizing noise is not to exceed $\pm x$ % of the peak-to-peak analog level. So, we have

$$|\text{error}| \le \frac{x}{100}(2m_p)$$

or $$\frac{m_p}{q} \le \frac{x}{100}(2m_p)$$

or $$\frac{1}{q} \le \frac{x}{50}$$

or $$q = 2^n \ge \frac{50}{x}$$

Thus, the required number of bits in each PCM word is

$$n \ge \log_2\left(\frac{50}{x}\right)$$

SOL 6.1.34 Option (C) is correct.

Given the absolute bandwidth of analog waveform,

$$W = 100 \text{ Hz}$$

So, the minimum sampling rate (Nyquist rate) for the waveform is obtained as

$$f_N = 2W = 2 \times 100 = 200 \text{ Hz}$$

SOL 6.1.35 Option (B) is correct.

We have the Nyquist rate for the waveform,

$$f_N = 200 \text{ Hz}$$

The peak-to-peak amplitude of message signal,

$$2m_p = 10 - (-10) = 20 \text{ V}$$

So, we have the maximum quantization error for the system,

$$|\text{error}| = \frac{\delta}{2} = \frac{2m_p}{2q} = \frac{m_p}{q}$$

Since, the accuracy for PCM system is $\pm 0.1\%$. So, we have

$$|\text{error}| \le 0.1\% \text{ of } (2m_p)$$

or $$\frac{m_p}{q} \le \frac{0.1}{100} \times 2m_p$$

or $$q \ge 500$$

or $$2^n \ge 500$$

or $$n \ge \log_2 500 = 8.96$$

i.e. the minimum number of bits per sample is 8.96. Approximating the value, we consider

$$n = 9 \text{ bits/samples}$$

Thus, the minimum bit rate for the transmitted signal is

$$\begin{aligned} R_b &= nf_s \\ &= (9 \text{ bits/sample}) \times (200 \text{ samples/sec}) \\ &= 1800 \text{ bits/sec} \end{aligned}$$

Therefore, the minimum absolute channel bandwidth required for transmission of the PCM is

$$\begin{aligned} B &= \frac{R_b}{2} = \frac{1800 \text{ bits/sec}}{2} \\ &= 900 \text{ Hz} \end{aligned}$$

SOL 6.1.36 Option (C) is correct.

Given, the sampling frequency of signal,

$$f_s = 8 \text{ k samples/sec} = 8 \text{ kHz}$$

Average SNR (signal to noise ratio) in the PCM system,

$$\left(\frac{S}{N}\right)_{\text{ave}} = 30 \text{ dB} = 10^3 = 1000 \qquad \text{...(1)}$$

In PCM system, we consider the noise due to quantization only. So, we have the average signal to quantization noise ratio defined as

$$\left(\frac{S}{N}\right)_{\text{ave}} = q^2 \qquad \text{...(2)}$$

From equations (1) and (2), we get

$$q^2 = 1000$$

or $$q = \sqrt{1000} = 31.6 \approx 32$$

This is the number of quantization level. So, we obtain the number of bits per sample as

$$n = \log_2 q = \log_2 32 = 5 \text{ bits/sample}$$

Therefore, the bit rate of the voice frequency signal is given by

$$R_b = nf_s = (5 \text{ bits/sample}) \times (8 \text{ ksamples/ sec})$$
$$= 5 \times 8 = 40 \text{ k bits/ sec}$$

Converting it into bytes/sec, we have

$$R_b = \frac{40}{8} \text{ k bytes/ sec} = 5 \text{ k bytes/ sec}$$

Since, an 820 Mbyte hard disk is used to store the PCM data, so the duration of stored voice frequency conversation is

$$T = \frac{850 \text{ Mbyte}}{5 \text{ k bytes/ sec}}$$
$$= \frac{850 \times 10^6}{5 \times 10^3} \text{ sec}$$
$$= 170 \times 10^3 \times \frac{1}{60} \text{ minute}$$
$$= 2.833 \times 10^3 \text{ minute}$$
$$= 2833 \text{ minute}$$

SOL 6.1.37 Option (A) is correct.

Given, the peak signal to quantizing noise ratio,

$$\left(\frac{S}{N}\right)_{\text{peak}} = 55 \text{ dB} = 10^{5.5}$$
$$3q^2 = 10^{5.5}$$
$$q = \sqrt{\frac{10^{5.5}}{3}} = 3.247 \times 10^2$$
$$n = \log_2 q = \log_2 (3.247 \times 10^2)$$
$$= 8.34$$

Since, we are solving for the least value of quantizing noise ratio, so we use

$$n = 9 \text{ bits}$$

Therefore, the required number of quantizing steps is

$$q = 2^n = 2^9 = 512$$

SOL 6.1.38 Option (A) is correct.

We have the required number of bits per sample,

$$n = 9 \text{ bits/sample}$$

Also, the bandwidth of analog signal,

$$W = 4.2 \text{ MHz}$$

So, we get the sampling frequency of the signal,

$$f_s = 2W = 2 \times (4.2) = 8.4 \text{ MHz}$$

Therefore, the bit rate of the signal is

$$R_b = nf_s$$
$$= (9 \text{ bits/sample}) \times (8.4\,M \text{ samples/ sec})$$
$$= 75.6\,M \text{ bit/ sec}$$

For rectangular pulse shape, the null channel bandwidth is obtained as

$$B_{\text{null}} = R_b = 75.6 \text{ MHz}$$

SOL 6.1.39 Option (C) is correct.

Given, the analog signal bandwidth

$$W = 20 \text{ kHz} = 20 \times 10^3 \text{ Hz}$$

Number of bits per sample,

$$n = 16 \text{ bits/sample}$$

Since, PCM uses 8 times oversampling of the analog signal, so we get the sampling frequency

$$\begin{aligned} f_s &= 8f_N = 8 \times (2W) \\ &= 8 \times (2 \times 20 \times 10^3) \\ &= 320 \text{ k samples/ sec} \end{aligned}$$

Thus, the null bandwidth of this PCM signal is given by

$$\begin{aligned} B_{\text{null}} &= R_b = nf_s \\ &= (16 \text{ bits/sample}) \times (320 \text{ k samples/ sec}) \\ &= 512 \text{ MHz} \end{aligned}$$

SOL 6.1.40 Option (C) is correct.

We have the number of bits per sample,

$$n = 16 \text{ bit}$$

Since, the PCM includes one parity bit, so we get the number of quantization level for the PCM as

$$q = 2^{n-1} = 2^{16-1} = 2^{15}$$

Therefore, the peak SNR for the signal is given by

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{peak}} &= 3q^2 \\ &= 3(2^{15})^2 = 3 \times 2^{30} \end{aligned}$$

In decibel, we get

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{peak}} &= 10\log_{10}(3 \times 2^{30}) \\ &= 95.08 \text{ dB} \end{aligned}$$

SOL 6.1.41 Option (A) is correct.

Given, the bit error rate due to channel noise (probability of bit error),

$$P_e = 10^{-4}$$

The peak signal to noise ratio on the recovered analog signal,

$$\left(\frac{S}{N}\right)_{\text{peak}} \geq 30 \text{ dB} = 10^3 = 1000 \quad \text{...(1)}$$

Again, for a PCM signal we have the peak signal to noise ratio,

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{peak}} &= \frac{3q^2}{1 + 4(q^2 - 1)P_e} \\ &= \frac{3 \times 2^{2n}}{1 + 4(2^{2n} - 1)10^{-4}} \quad \text{...(2)} \end{aligned}$$

where n is number of bits per sample. Equations (1) and (2) can be solved for the value of n by hit and trial method. Firstly, we put $n = 4$ in equation (2)

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{peak}} &= \frac{3 \times 2^{2\times4}}{1 + 4(2^{2\times4} - 1)10^{-4}} \\ &= 6.969 \times 10^2 \\ &= 10\log_{10}(6.969 \times 10^2) \text{ dB} \\ &= 28.4 \text{ dB} \end{aligned}$$

Again, for $n = 5$ in equation (2) we obtain

$$\left(\frac{S}{N}\right)_{\text{peak}} = \frac{3 \times 2^{2\times5}}{1 + 4(2^{2\times5} - 1)10^{-4}}$$

$$= 2.1799 \times 10^3$$
$$= 10\log_{10}(2.1799 \times 10^3)\text{ dB}$$
$$= 33.4\text{ dB}$$

Thus, $n = 5$ satisfies the required condition. Therefore, the minimum number of quantizing steps is

$$q = 2^n = 2^5 = 32$$

SOL 6.1.42 Option (A) is correct.

Given, the bandwidth of the original analog signal,

$$W = 2.7\text{ kHz}$$

So, we get the sampling frequency of the PCM signal,

$$f_s = 2W = 2 \times 2.7\text{ kHz}$$
$$= 5.4\text{ k samples/ sec}$$

Also, we have number of bits per sample, $n = 5$ bits/sample.

Therefore, the null bandwidth of the PCM signal is

$$B = R_b = nf_s$$
$$= 5 \times (5.4\text{ k samples/ sec})$$
$$= 27\text{ kHz}$$

SOL 6.1.43 Option (C) is correct.

Given, the number of possible levels,

$$M = 16\text{ levels}$$

So, the number of bits corresponding to each level is obtained as

$$k = \log_2 M = \log_2 16$$
$$= 4\text{ bits/level}$$

SOL 6.1.44 Option (C) is correct.

Given that the digital communication system sends one level over the channel every 0.8 ms. so, we get the baud rate (symbol rate) of the signal as

$$D = \frac{1}{T_s} = \frac{1}{0.8 \times 10^{-3}}$$
$$= 1.25 \times 10^3$$
$$= 1250\text{ baud}$$
$$= 1250\text{ levels/ sec}$$

SOL 6.1.45 Option (A) is correct.

We have the baud rate of symbol,

$$D = 1250\text{ levels/ sec}$$

Number of bits per level,

$$k = 4\text{ bits/level}$$

So, we get the bit rate for the signal as

$$R_b = kD$$
$$= (4\text{ bits/level}) \times (1250\text{ levels/ sec})$$
$$= 5000\text{ bits/ sec}$$
$$= 5\text{ k bits/ sec}$$

SOL 6.1.46 Option (B) is correct.

Given, the bandwidth of analog signal

$$W = 2700\text{ Hz}$$

So, we have the sampling frequency,

$$f_s = 2W = 2 \times 2700$$
$$= 5400 \text{ Hz} = 5.4 \text{ kHz}$$

Now, accuracy at the output is $\pm 1\%$ of full scale V_{PP}. So, we have the step size of quantization.

$$\delta \le 2 \times \frac{1}{100} V_{PP}$$
$$\frac{V_{PP}}{2^n} \le 2 \times \frac{1}{100} V_{PP}$$
$$n \ge \log_2 \frac{100}{2} = 5.64$$

Therefore, we use $n = 6$. Thus, the minimum bit rate of the PCM signal is obtained as

$$R_b = nf_s = 6 \times 5.4$$
$$= 32.4 \text{ k bits/ sec}$$

SOL 6.1.47 Option (D) is correct.

We have the minimum bit rate of PCM signal as

$$R_b = 32.4 \text{ k bits/ sec}$$

This signal is converted to $M = 8$ levels signals. So, we have the number of bits per level (symbol) as

$$k = \log_2 M = \log_2 8 = 3 \text{ bits/symbol}$$

Therefore, we obtain the baud rate (symbol/sec) as

$$D = \frac{R_b}{k}$$
$$= \frac{32.4 \text{ k bits/ sec}}{3 \text{ bits/symbol}}$$
$$= 10.8 \text{ k symbol/ sec}$$

SOL 6.1.48 Option (B) is correct.

We have the baud rate of signal,

$$D = 10.8 \text{ k symbols/ sec}$$

So, we obtain the minimum absolute channel bandwidth required for transmission of PCM signal as

$$B = \frac{D}{2} = \frac{10.8}{2}$$
$$= 5.4 \text{ kHz}$$

SOL 6.1.49 Option (C) is correct.

Given, the bit rate of binary waveform,

$$R_b = 9600 \text{ bits/ sec}$$

This waveform is converted into octal waveform, so we have the number of levels,

$$M = 8$$

Therefore, the number of bits per level is obtained as

$$k = \log_2 8 = 3 \text{ bits/symbol}$$

Thus, the baud rate of the multilevel signal is obtained as

$$D = \frac{R_b}{k} = \frac{9600 \text{ bits/ sec}}{3 \text{ bits/symbol}}$$
$$= 3.2 \text{ k symbol/ sec}$$

SOL 6.1.50 Option (B) is correct.

We have the absolute bandwidth of the system,

$$B = 2.4\text{ kHz}$$

Also, the baud rate is

$$D = 3.2\text{ k symbols/ sec}$$

Since, the baud rate for a system is defined as

$$D = \frac{2B}{1+\alpha}$$

where α is the roll off factor. Substituting the given values, we obtain the roll off factor of the filter characteristic as

$$3.2 \times 10^3 = \frac{2 \times 2.4 \times 10^3}{1+\alpha}$$

or $$\alpha = \frac{1}{2} = 0.5$$

SOL 6.1.51 Option (A) is correct.

Given, the channel bandwidth

$$B = 4\text{ kHz}$$

Since the PCM signal is a binary polar NRZ line code $(M = 2)$, so we have number of bits per symbol.

$$k = 1\text{ bits/symbol}$$

Therefore, the maximum PCM bit rate is obtained as

$$R_b = kD = D = \frac{2B}{1+\alpha}$$

where D is the baud rate (symbols/sec) and α is the roll off factor. So, we get

$$R_b = \frac{2 \times 4\text{ kHz}}{1+0.5} = 5.33\text{ k bits/ sec}$$

This is the maximum PCM bit rate that can be supported by this system without introducing ISI.

SOL 6.1.52 Option (A) is correct.

We have the maximum PCM bit rate, $R_b = 5.33\text{ k bits/ sec}$

Number of quantization level, $q = 16$

So, we get the number of bits per sample for the PCM signal as

$$n = \log_2 q = \log_2 16 = 4$$

Therefore, the sampling frequency for the signal is given by

$$f_s = \frac{R_b}{n}$$

$$= \frac{5.33 \times 10^3}{4} = 1.33\text{ kHz}$$

Thus, the maximum bandwidth of analog signal is

$$W = \frac{f_s}{2} = \frac{1.33}{2} = 0.666\text{ kHz} = 666\text{ Hz}$$

SOL 6.1.53 Option (D) is correct.

Given, the bit rate of data,

$$R_b = 2400\text{ bits/ sec}$$

This data is transmitted over a channel using $M = 4$ level line code. So, we get the number of bits per level,

$$k = \log_2 M = \log_2 4 = 2\text{ bits/symbol}$$

Therefore, the baud rate is given by

$$\begin{aligned} D &= \frac{R_b}{k} \\ &= \frac{2400 \text{ bits/ sec}}{2 \text{ bits/symbol}} \\ &= 1200 \text{ symbols/ sec} \end{aligned}$$

Thus, the 6 dB bandwidth of the channel is obtained as

$$\begin{aligned} B_{6\,\text{dB}} &= \frac{D}{2} = \frac{1200 \text{ symbols/ sec}}{2} \\ &= 600 \text{ Hz} \end{aligned}$$

SOL 6.1.54 Option (A) is correct.

We have the baud rate (symbol rate) for the system as

$$D = 1200 \text{ symbols/sec}$$

Also, the roll off factor for the Nyquist characteristic is

$$\alpha = 0.5$$

So, we get the absolute bandwidth for the system as

$$\begin{aligned} B_{\text{absolute}} &= \frac{1}{2}(1+\alpha)D \\ &= \frac{1}{2}(1+0.5) \times 1200 \\ &= 900 \text{ Hz} \end{aligned}$$

SOL 6.1.55 Option (A) is correct.

Given, the frequency of message signal, $f_m = 10 \text{ kHz}$

Peak-to-peak amplitude of the test signal, $2A_m = 1 \text{ volt}$

So, we have the Nyquist rate for the signal, $f_N = 2f_m = 2 \times 10 = 20 \text{ kHz}$

Since, the signal is sampled at 10 times the Nyquist rate, so we get the sampling frequency

$$f_s = 10f_N = 10 \times 20 = 200 \text{ kHz}$$

To avoid the slope overload, we have

$$\begin{aligned} \delta f_s &\geq \max\left|\frac{dm(t)}{dt}\right| \\ \delta \times 200 \text{ kHz} &\geq 2\pi f_m A_m \\ \delta &\geq \frac{2\pi \times (10 \text{ kHz}) \times \left(\frac{1}{2}\right)}{200 \text{ kHz}} \\ \delta &\geq 0.157 \text{ volt} \end{aligned}$$

SOL 6.1.56 Option (D) is correct.

For a delta modulation system, the power spectral density of granular noise is obtained as

$$\begin{aligned} S_N(f) &= \frac{\delta^2}{6f_s} = \frac{(0.157)^2}{6 \times 200 \text{ kHz}} \\ &= 2.06 \times 10^{-8} \text{ V}^2/\text{Hz} \end{aligned}$$

SOL 6.1.57 Option (A) is correct.

We have the power spectral density, of granular noise

$$S_N(f) = 2.06 \times 10^{-8} \text{ V}^2/\text{Hz}$$

Bandwidth of sampled signal,

$$B = f_s = 200 \text{ kHz}$$

So, we have the total noise at receiver output as

$$\begin{aligned} P_{no} &= S_N(f) \times 2B \\ &= (2.06 \times 10^{-8}) \times (2 \times 200 \times 10^3) \\ &= 8.24 \times 10^{-3} \text{ W} \end{aligned}$$

Also, we have the maximum amplitude of test signal,

$$A_m = \frac{1}{2} \text{ volt}$$

Therefore, the signal power at the receiver output is

$$P_{so} = \frac{A_m^2}{2} = 0.125 \text{ W}$$

Thus, the signal to quantization noise ratio at receiver output is

$$\begin{aligned} \left(\frac{S}{N}\right)_0 &= \frac{P_{so}}{P_{no}} \\ &= \frac{0.125}{8.24 \times 10^{-3}} = 15.2 \text{ W/W} \end{aligned}$$

In decibel, we get

$$\begin{aligned} \left(\frac{S}{N}\right)_{\text{dB}} &= 10\log(15.2) \\ &= 11.81 \text{ dB} \end{aligned}$$

SOL 6.1.58 Option (D) is correct.

Given, the bandwidth of five message signals, respectively as

$$W, W, 2W, 4W, 4W$$

Since, the bandwidths of message signals are harmonically related, so we have the minimum transmission bandwidth for TDM signal as

$$\begin{aligned} B &= W + W + 2W + 4W + 4W \\ &= 12W \end{aligned}$$

****

SOLUTIONS 6.2

SOL 6.2.1 Correct answer is 50.

Given, the bandwidth of each signal, $f = 1\,\text{kHz}$

From the Nyquist criterion the sampling rate must be such that $f_s \geq 2f$

Here, the period T_s of $c(t)$ is chosen for the maximum allowable value. So, we have the sampling frequency (frequency of pulse train)

$$f_s = 2f = 2 \times 10^3 \text{ Hz}$$

Therefore, the time period of pulse train is given by

$$T_s = \frac{1}{f_s} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3} \text{ sec}$$

After sampling of the multiplexed signals (10 signals), each of the signal will appear for time duration $\triangle$. So, for the largest value of $\triangle$, we must have

$$10\,\triangle = T_s$$

$$\triangle = \frac{T_s}{10} = \frac{0.5 \times 10^{-3}}{10} = 5 \times 10^{-5}\text{ s} = 50\,\mu\text{s}$$

SOL 6.2.2 Correct answer is 317.5.

Given the sampling rate of a signal,

$$f_s = 44.1\,\text{kHz} = 44.1 \times 10^3 \text{ samples/sec}$$

Since, recording system samples the signal with 16 bit analog to digital converter. Therefore, the rate of recording in bits/second is given by

$$f_s = (44.1 \times 10^3) \times 16 \text{ bits/sec}$$
$$= 7.056 \times 10^5 \text{ bits/sec}$$

As the CD can record an hour worth of music, the approximate capacity of CD is given by

$$\text{Memory capacity} = f_s \times \text{Time duration}$$
$$= (7.056 \times 10^5) \times (3600\text{ sec})$$
$$= 2.54 \times 10^9 \text{ bits}$$

Converting it into bytes (1 byte = 8 bites), we get

$$\text{Memory capacity} = \frac{2.54 \times 10^9}{8} \text{ bytes}$$
$$= 3.175 \times 10^8 \text{ bytes}$$
$$= 317.5 \text{ M bytes}$$

SOL 6.2.3 Correct answer is 3.47.

Given, the sampling frequency $f_s = 36\,\text{kHz} = 36 \times 10^3\,\text{Hz}$

Number of quantization levels, $q = 256$

Let these quantization levels be encoded in binary by n bits. So, we have

$$2^n = q = 256$$

or $$n = 8$$

So, there are 8 bits to represent a sample. The time duration of a sample is given by

$$T_s = \frac{1}{f_s} = \frac{1}{36 \times 10^3} = 2.78 \times 10^{-5} \text{ sec}$$

Therefore, the time duration of a bit of the binary coded signal is obtained as

$$T_b = \frac{T_s}{8} = \frac{2.78 \times 10^{-5}}{8}$$
$$= 3.47 \times 10^{-6} \text{ sec}$$
$$= 3.47 \,\mu\text{s}$$

SOL 6.2.4 Correct answer is 7.

Given, the tolerable error in sample amplitude $= 0.5\%$ of peak-peak full scale.

Let the analog signal lie in the internal $(-m_p, m_p)$; where m_p is the peak value of the signal. If q levels are used to quantize the signal, then the step size of each level is

$$\delta = \frac{2m_p}{q}$$

So, the maximum quantization error in the signal is obtained as

$$\text{error} = \frac{\delta}{2} = \frac{2m_p}{2q} = \frac{m_p}{q} \qquad ...(1)$$

Also, we have the tolerable error in sample amplitude

$$\text{error} = 0.5\% \text{ of peak-peak full scale}$$
$$= \frac{0.5}{100} \times (2m_p)$$
$$= 0.01\, m_p \qquad ...(2)$$

As the maximum quantization error of the signal must be less than the tolerable error, so from equations (1) and (2), we have

$$\frac{m_p}{q} \le 0.01\, m_p$$

or, $$q \ge 100$$

Let the minimum number of binary digits required to encode the sample be n.

Then, we have

$$2^n \ge 100$$

The minimum value of n that satisfies the above condition is

$$n = 7$$

SOL 6.2.5 Correct answer is 144.

Given the bandwidth of signal, $f_m = 6\,\text{MHz} = 6 \times 10^6$ Hz

Number of quantization levels, $q = 1024$

So, we have the Nyquist rate for the signal as

$$f_N = 2f_m$$
$$= 2 \times 6 \times 10^6$$
$$= 12 \times 10^6 \text{ Hz.}$$

Since, the sampling rate must be 20% above the Nyquist rate therefore, we get the sampling rate as

$$f_s = 1.2 \times f_N$$
$$= 1.2 \times (12 \times 10^6)$$
$$= 1.44 \times 10^7 \text{ Hz.}$$

Again, let the number of bits for quantization be n. So, we have

$$q = 2^n = 1024$$

or, $$n = 10 \text{ bits}$$

So, the bit rate of the sampled signal is given by

$$\begin{aligned} R_b &= nf_s \\ &= 10 \times (1.44 \times 10^7) \\ &= 1.44 \times 10^8 \text{ bits/sec} \\ &= 144 \text{ Mbits/sec} \end{aligned}$$

Therefore, the maximum transmission bandwidth of the signal is

$$B_T = R_b = 144 \text{ Mbits/sec.}$$

This is the minimum channel bandwidth required to transmit the signal.

NOTE :

The maximum transmission bandwidth is defined for the case when one bit information is transmitted in a specific instant so, we have the maximum transmission bandwidth

$$B_T = R_b$$

SOL 6.2.6 Correct answer is 450.

Given, the audio signal bandwidth, $f_m = 15$ kHz

Number of quantization levels, $q = 32678$

So, we have the Nyquist sampling rate for the analog signal,

$$\begin{aligned} f_s &= 2f_m \\ &= 2 \times 15 = 30 \text{ kHz} \end{aligned}$$

Let n number of bits encode each sample, then we have

$$q = 2^n = 32678$$

or, $$n = 15 \text{ bits}$$

Therefore, the minimum bit rate for the analog signal is

$$\begin{aligned} R_b &= nf_s \\ &= 15 \times 30 \\ &= 450 \text{ kbits/s} \end{aligned}$$

SOL 6.2.7 Correct answer is 7.

Given the rate of generation of characters

$$R_c = 1{,}000{,}000 \text{ characters/sec}$$

Total number of characters in the American standard code

$$q = 128 \text{ characters}$$

Let the minimum no. of bits required to encode these characters be n.

So, we have

$$q = 2^n = 128$$

or, $$n = 7 \text{ bits}$$

So, 7 bits are required to encode each character. Therefore, we have the minimum bit rate for the signal,

$$\begin{aligned} R_b &= nR_c \\ &= 7 \times 1{,}000{,}000 \\ &= 7{,}000{,}000 \text{ bits/second} \\ &= 7 \text{ Mbits/sec} \end{aligned}$$

So, the transmission bandwidth of the signal is

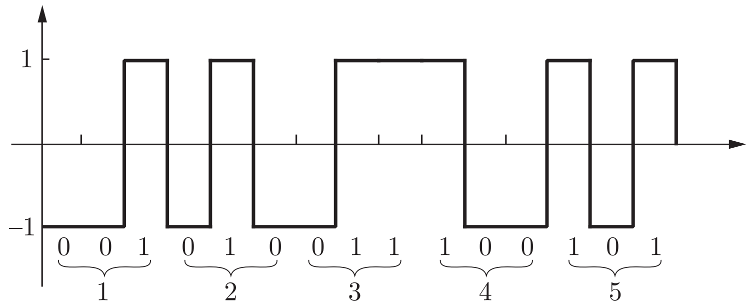
$$B_T = R_b = 7 \text{ Mbits/sec.}$$

This is the minimum channel bandwidth required to transmit the signal.

SOL 6.2.8

Correct answer is 12345.

Observing the PCM signal, we obtain the transmitted bits as shown below



In the figure, we have put 0 for level of -1 volt and 1 for level of $+1$ volt. Since, the code word used consists of 3 bits, so, we write the words for the transmitted bits as

1st word $\rightarrow$ 001 $\rightarrow$ 1

2nd word $\rightarrow$ 010 $\rightarrow$ 2

3rd word $\rightarrow$ 011 $\rightarrow$ 3

4th word $\rightarrow$ 100 $\rightarrow$ 4

5th word $\rightarrow$ 101 $\rightarrow$ 5

Thus, the sampled version of analog signal from which this PCM signal is driven is 12345

SOL 6.2.9

Correct answer is 6.25.

Given the number of bits of encoder, $n = 8$ bits.

Bit rate of the system, $R_b = 10^8$ bits/sec

Let the maximum message bandwidth be W_m. So, we have the sampling frequency for this signal as $f_s = 2W_m$.

Therefore, the bit rate for the signal is obtained as

$$\begin{aligned} R_b &= nf_s \\ 10^8 &= 8 \times (2W_m) \\ W_m &= \frac{10^8}{16} \\ &= 6.25 \times 10^6 \text{ Hz} \\ &= 6.25 \text{ MHz} \end{aligned}$$

SOL 6.2.10

Correct answer is 4.

Given, the sampling rate, $f_s = 8\,\text{kHz}$

ON time duration of the flat top samples, $\triangle = 1\,\mu\text{s}$

The time duration for synchronization, $T_{syn} = 1\,\mu\text{s}$

Since, each of the 24 voice signals are sampled for the time duration $\triangle$. So, the total time width for the 24 voice signal pulses is given by

$$\begin{aligned} T_{voice} &= 24\triangle \\ &= 24 \times 1 = 24\,\mu\text{s} \end{aligned}$$

Therefore, the total time width required for voice signal pulses along with its synchronization pulse is

$$\begin{aligned} T_{pulse} &= T_{voice} + T_{syn} \\ &= 24 + 1 \\ &= 25\,\mu\text{s} \end{aligned}$$

Again, we have the time duration of each sample as

$$T_s = \frac{1}{f_s} = \frac{1}{8 \times 10^3} = 1.25 \times 10^{-4} \text{ sec}$$

$$= 125\,\mu\text{s}$$

Therefore, the total OFF duration in a sample is

$$T_{\text{OFF}} = T_s - T_{\text{pulse}}$$
$$= 125 - 25$$
$$= 100\,\mu\text{s}$$

Since, there are 25 pulses (24 voice signal pulses and 1 synchronization pulse) in a sample. So, the spacing between successive pulses is given by

$$T_{\text{spacing}} = \frac{T_{\text{OFF}}}{25}$$
$$= \frac{100}{25} = 4\,\mu\text{s}$$

ALTERNATIVE METHOD :

Given, the sampling frequency $f_s = 8\text{ kHz}$

So, we have the time duration of a sample,

$$T_s = \frac{1}{f_s}$$
$$= \frac{1}{8 \times 10^3} = 1.25 \times 10^{-4}\text{ sec}$$
$$= 125\,\mu\text{s}$$

Now, we have the 24 voice signals each sampled for $1\,\mu\text{s}$ duration and one synchronization pulse of $1\,\mu\text{s}$ duration. So, the total number of pulses in a sample is 25. Therefore, the alloted time duration to each of the pulse is

$$T_{s,\text{pulse}} = \frac{T_s}{25} = \frac{125}{25} = 5\,\mu\text{s}$$

Since the width of each pulse is $1\,\mu\text{s}$. Therefore, the spacing between the successive pulses is

$$T_{\text{spacing}} = T_{s,\text{ pulse}} - 1\,\mu\text{s}$$
$$= 5 - 1$$
$$= 4\,\mu\text{s}$$

SOL 6.2.11 Correct answer is 1.0823.

Given, the bandwidth of speech signal, $W_m = 3.4\text{ kHz}$

So, the Nyquist rate of speech signal is $f_N = 2W_m = 2 \times 3.4 = 6.8\text{ kHz}$

Since, the sampling rate is 10 times the Nyquist rate so, we have

$$f_s = 10 f_N = 10 \times 6.8 = 6.8\text{ kHz.}$$

Also, we have the step size for the delta modulated signal $\delta = 100\text{ mV}$

To avoid the slope overload for this delta modulator, we have the condition

$$\frac{\delta}{T_s} \geq \max\left|\frac{dm(t)}{dt}\right|$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq \frac{100 \times 10^{-3}}{1/(68 \times 10^3)}$$

or, $$\max\left|\frac{dm(t)}{dt}\right| \leq 68 \times 10^2$$

Now, we have the test signal with 1 kHz frequency and let the maximum amplitude of this test signal be A_m. therefore, substituting it in above condition, we get

$$A_m(2\pi f_m) \leq 68 \times 10^2$$

or $$A_m \leq \frac{68 \times 10^2}{2\pi \times 10^3} = 1.0823$$

Hence, $A_m = 1.0823$ is the maximum amplitude of test signal.

SOL 6.2.12

Correct answer is 20%.

Given the output signal to quantization noise ratio of $n_1 = 10$ bit PCM,

$$(\mathrm{SNR})_1 = 30dB = 10^{30/10} = 10^3$$

After increasing the number of quantization level, the desired SNR

$$(\mathrm{SNR})_2 = 42\text{ dB} = 10^{42/10} = 10^{4.2}$$

Since, SNR for a PCM signal is given by

$$\frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p^2}$$

where $\overline{m^2(t)}$ is the power of message signal and m_p is its peak value. So, we can write

$$\frac{S_o}{N_o} \propto q^2$$

or, $$\mathrm{SNR} = kq^2$$

where k is a constant for the given message signal. Here, the message signal for both the cases are same, so the value of k will be constant for both the cases, whereas the value of q varies.Therefore, the ratio of SNR is

$$\frac{(\mathrm{SNR})_1}{(\mathrm{SNR})_2} = \frac{q_1^2}{q_2^2} \qquad ...(1)$$

Initially, the quantization level is $q_1 = 2^{n_1} = 2^{10}$

For the desired value of SNR, let the system require n_2 bits. So, we have

$$q_2 = 2^{n_2}$$

Substituting all the values in equation (1), we get

$$\frac{10^3}{10^{4.2}} = \left(\frac{2^{10}}{2^{n_2}}\right)^2$$

$$10^{\frac{4.2-3}{2}} = 2^{n_2-10}$$

$$n_2 - 10 = \log_2 10^{0.6}$$

$$n_2 = 10 + 0.6\log_2 10$$

Thus, the increase in bits of PCM is

$$n_2 - n_1 = (10 + 0.6\log_2 10) - 10$$

$$= 0.6\log_2 10$$

Therefore, the percentage increase in the transmission bandwidth is given by

$$\%B_T = \frac{n_2 - n_1}{n_1} \times 100\%$$

$$= \frac{0.6\log_2 10}{10} \times 100\%$$

$$= 20\%$$

NOTE :

Since, the message signal (sampling frequency) for both the cases are same so, the percentage increase in transmission bandwidth is same as the percentage increase in number of encoding bits.

SOL 6.2.13

Correct answer is 10.

Given, the compression parameter of PCM signal, $\mu = 100$

Minimum signal to quantization noise ratio,

$$(\mathrm{SNR})_{\min} = 50\text{ dB} = 10^5$$

For a PCM system, the output SNR (for μ-law companding) is given by

$$\frac{S_o}{N_o} = \frac{3q^2}{[\ln(1+\mu)]^2}$$

where q is the number of quantization level defined as

$$q = 2^n$$

where n is the number of bits required to encode the sample. So, we have

$$\frac{S_o}{N_o} = \frac{3(2^n)^2}{[\ln(1+100)]^2} > 10^5$$

$$\frac{3\times 2^{2n}}{[\ln(101)]^2} > 10^5$$

$$2^{2n} > 7.1\times 10^5$$

$$n > \frac{1}{2}\log_2(7.1\times 10^5)$$

$$n > 9.72$$

Thus, the minimum number of bits per sample is

$$n = 10$$

SOL 6.2.14 Correct answer is 8.

Given, the signal to quantization ratio for the PCM,

$$\frac{S_o}{N_o} = 48\text{ dB} = 10^{4.8}$$

Now, the signal to quantization noise ratio for a PCM system is defined as

$$\frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p^2} \qquad ...(1)$$

where $\overline{m^2(t)}$ is the power in the message signal $m(t)$ and m_p is its peak value. Here, the message signal is sinusoid, so we have

$$\overline{m^2(t)} = \frac{1}{2}$$

and $m_p = 1$

Substituting it in equation (1), we have

$$\frac{S_o}{N_o} = 3q^2\frac{\frac{1}{2}}{1}$$

$$10^{4.8} = \frac{3q^2}{2}$$

$$q^2 = \frac{2\times 10^{4.8}}{3} \qquad ...(2)$$

Now, if the number of bits per sample of PCM be n, then we must have

$$q < 2^n$$

Substituting it in equation (2), we get

$$(2^n)^2 > \frac{2\times 10^{4.8}}{3}$$

or, $$n > \frac{1}{2}\log_2\left(\frac{2\times 10^{4.8}}{3}\right)$$

or, $$n > 7.7$$

Thus, the minimum number of bits per sample is $n = 8$

SOL 6.2.15 Correct answer is 140.

Given the duration of speech signal, $T = 20$ sec.

Sampling frequency of the signal, $f_s = 8\text{ kHz}$

Signal to quantization noise ratio,

$$\frac{S_o}{N_o} = 40\text{ dB} = 10^4$$

Now, for a speech signal (sinusoid signal) we have the signal to quantization noise ratio

$$\frac{S_o}{N_o} = 3q^2\frac{\overline{m^2(t)}}{m_p}$$

$$= 3q^2\frac{1/2}{1} = \frac{3q^2}{2}$$

If n be the minimum number of bits per sample so, we have

$$\frac{3q^2}{2} > 10^4$$

or $$\frac{3(2^{2n})^2}{2} > 10^4$$

or, $$n > \frac{1}{2}\log_2\left(\frac{2\times10^4}{3}\right) = 6.35$$

So, we have

$$n = 7$$

Therefore, the data rate (bit rate) of the speech signal is given by

$$R_b = nf_s$$

$$= 7\times 8 \text{ kHz}$$

$$= 56 \text{ kbits/sec}$$

So, the minimum storage capacity needed to accommodate the 20 sec duration speech signal is

$$M = R_b \times T$$

$$= (56 \text{ kbits/ sec})\times(20 \text{ sec})$$

$$= 1120 \text{ kbits}$$

Converting it into bytes (1 byte = 8 bits), we get

$$M = \frac{1120}{8} = 140 \text{ k bytes.}$$

SOL 6.2.16 Correct answer is 12.

Given, the peak to peak amplitude of the message signal $m(t)$,

$$2m_p = 1.536 \text{ V or } m_p = 0.768 \text{ V}$$

number of quantization level, $q = 128$

So, the quantization noise power for the uniform mid rise quantized signal $m(t)$ is given by

$$N_o = \frac{m_p^2}{3q^2}$$

$$= \frac{(0.768)^2}{3\times(128)^2}$$

$$= 1.2\times10^{-5}\text{ V}^2 = 12\times10^{-6}\text{ V}^2$$

SOL 6.2.17 Correct answer is 0.0014.

Given the message signal,

$$m(t) = \sin c(700t) + \sin c(500t)$$

$$= \frac{\sin(700\pi t)}{\pi t} + \frac{\sin(500\pi t)}{\pi t}$$

$$= \frac{1}{\pi t}[\sin(700\pi t) + \sin(500\pi t)]$$

So, we have the frequency components of the signal as

$$f_1 = \frac{700\pi}{2\pi} = 350 \text{ Hz}$$

$$f_2 = \frac{500\pi}{2\pi} = 250 \text{ Hz}$$

Therefore, the signal is band limited to $f_m = 350$ Hz.

So, the Nyquist sampling rate for the signal is given by

$$f_N = 2f_m = 2\times(350) = 700 \text{ Hz.}$$

Therefore, the Nyquist sampling interval is obtained as

$$T_N = \frac{1}{f_N} = \frac{1}{700} = 0.0014 \text{ sec.}$$

SOL 6.2.18 Correct answer is 16.

For a PCM system, the signal to quantization noise ratio is defined as

$$\text{SNR} = 3q^2\frac{\overline{m^2(t)}}{m_p^2} = 3\times(2)^{2n}\frac{\overline{m^2(t)}}{m_p^2}$$

where n is the number of bits per sample, $\overline{m^2(t)}$ is the power in message signal, and m_p is the peak amplitude of $m(t)$.

When the code word length (n) is changed from $n_1 = 6$ to $n_2 = 8$ bits the signal to quantization noise ratio improves by the factor given as

$$\frac{(\text{SNR})_2}{(\text{SNR})_1} = \frac{3\times(2)^{2n_2}}{3\times(2)^{2n_1}} = \frac{3\times(2)^{2\times8}}{3\times(2)^{2\times6}} = 16$$

SOL 6.2.19 Correct answer is 384.

Given, the bandwidth of signals $g_1(t)$ and $g_4(t)$

$$f_1 = f_4 = 4 \text{ kHz}$$

Bandwidth of signals $g_2(t)$ and $g_3(t)$.

$$f_2 = f_3 = 8 \text{ kHz}$$

Required number of bits for each sample, $n = 8$ bits

For the given signals, we obtain the minimum sampling frequency (Nyquist sampling rate) as

$$f_{s1} = f_{s4} = 2\times4 = 8 \text{ kHz}$$

and $$f_{s2} = f_{s3} = 2\times8 = 16 \text{ kHz}$$

So, total number of pulses (frequency) per second for the multiplexed signal is

$$\begin{aligned} f_s &= f_{s1}+f_{s2}+f_{s3}+f_{s4} \\ &= 8+16+16+8 \\ &= 48 \text{ kHz} \end{aligned}$$

Therefore, the minimum transmission bit rate for the system is obtained as

$$\begin{aligned} R_b &= nf_s \\ &= 8\times48 \\ &= 384 \text{ k bits/sec} \end{aligned}$$

SOL 6.2.20 Correct answer is 57.6.

Given, the bandwidths of three analog signals,

$$f_1 = 1200 \text{ Hz}$$
$$f_2 = 600 \text{ Hz}$$
$$f_3 = 600 \text{ Hz}$$

Number of bits per sample, $n = 12$ bits

Since, the analog signals are sample at their Nyquist rate. So, we have their sampling frequencies as

$$f_{s_1} = 2f_1 = 2\times1200 = 2400 \text{ Hz}$$
$$f_{s_2} = 2f_2 = 2\times600 = 1200 \text{ Hz}$$
$$f_{s_3} = 2f_3 = 2\times600 = 1200 \text{ Hz}$$

So, we get the frequency (total number of pulses per second) for the multiplexed signal as

$$\begin{aligned} f_s &= f_{s_1} + f_{s_2} + f_{s_3} \\ &= 2400 + 1200 + 1200 \\ &= 4800 \text{ Hz} \end{aligned}$$

Therefore, the bit rate of the multiplexed signal is

$$\begin{aligned} R_b &= nf_s \\ &= 12 \times 4800 \\ &= 57.6 \text{ kbits/sec} \end{aligned}$$

SOL 6.2.21 Correct answer is 80.

Given, the bandwidth of each of the four signals,

$$f_m = 5 \text{ kHz}$$

Since, the signals to be sampled at twice the Nyquist rate. So, we have the sampling frequency of the signals as

$$\begin{aligned} f_s &= 2f_N = 2(2f_m) \\ &= 2 \times 2 \times 5 = 20 \text{ kHz} \end{aligned}$$

Now, these signals are time division multiplexed. So, the total pulses per second (frequency) of the multiplexed signal is

$$f_T = 4 \times f_s = 4 \times 20 = 80 \text{ kHz}$$

This is the theoretical transmission bandwidth of the channel.

SOL 6.2.22 Correct answer is 1600.

Given, the bandwidths of four independent message signals.

$$\begin{aligned} f_1 &= 100 \text{ Hz} \\ f_2 &= 100 \text{ Hz} \\ f_3 &= 200 \text{ Hz} \\ f_4 &= 400 \text{ Hz} \end{aligned}$$

Since, these signals are sampled at their Nyquist rate so, we have the sampling frequencies of the four signals as

$$\begin{aligned} f_{s1} &= 2f_1 = 2 \times 100 = 200 \text{ Hz} \\ f_{s2} &= 2f_2 = 2 \times 100 = 200 \text{ Hz} \\ f_{s3} &= 2f_3 = 2 \times 200 = 400 \text{ Hz} \\ f_{s4} &= 2f_4 = 2 \times 400 = 800 \text{ Hz} \end{aligned}$$

Now, these signals are time division multiplexed. So, the total pulses per second (frequency) of the multiplexed signal is

$$\begin{aligned} f_s &= f_{s1} + f_{s2} + f_{s3} + f_{s4} \\ &= 200 + 200 + 400 + 800 = 1600 \text{ Hz} \end{aligned}$$

This is the transmitted sample rate.

SOL 6.2.23 Correct answer is 350.

Given the message signal

$$\begin{aligned} g(t) &= 10\cos(50\pi t)\cos^2(150\pi t) \\ &= 10\cos(50\pi t)\left[\frac{1+\cos(2 \times 150\pi t)}{2}\right] \\ &= \frac{10}{2}\cos(50\pi t)[1+\cos(300\pi t)] \\ &= 5\cos(50\pi t) + 5\cos(50\pi t)\cos(300\pi t) \\ &= 5\cos(50\pi t) + \frac{5}{2}[\cos(250\pi t) + \cos(350\pi t)] \end{aligned}$$

So, the message signal is band limited to

$$f_m = \frac{350\pi}{2\pi} = 175 \text{ Hz}$$

Therefore, the Nyquist sampling rate for the signal is

$$f_N = 2f_m$$
$$= 2 \times 175 = 350 \text{ Hz}$$

SOL 6.2.24 Correct answer is 1280.

Given the sampling rate of one channel

$$f_{ch} = 8000 \text{ times/sec}$$
$$= 8000 \text{ Hz}$$

Since, 20 such channels are timedivision multiplexed so, we get the total pulse per second (frequency) of the multiplexed signal as

$$f_s = 20f_{ch}$$
$$= 20 \times 8000$$
$$= 160 \text{ kHz}$$

Now, each of the sample is represented by 7 bits and contains an additional bit for synchronization. So, the total number of bits per sample is obtained as

$$n = 7 + 1 = 8 \text{ bits}$$

Therefore, the total bit rate of TDM link is given by

$$R_b = nf_s$$
$$= 8 \times 160 \text{ kHz}$$
$$= 1280 \text{ kbits/sec}$$

SOL 6.2.25 Correct answer is 0.0029.

Given, the signal

$$s(t) = \sin c(350t) + \sin c(250t)$$
$$= \frac{\sin(350\pi t)}{\pi t} + \frac{\sin(250\pi t)}{\pi t}$$
$$= \frac{1}{\pi t}[\sin(350\pi t) + \sin(250\pi t)]$$

So, the signal is band limited to

$$f_m = \frac{350\pi}{2\pi} = 175 \text{ Hz}$$

Therefore, the Nyquist sampling rate for the signal is

$$f_N = 2f_m = 2 \times 175 = 350 \text{ Hz}$$

So, the Nyquist sampling internal is given by

$$T_N = \frac{1}{f_N} = \frac{1}{350} = 0.0029 \text{ sec}$$

SOL 6.2.26 Correct answer is 4233.6.

Given the sampling frequency, $f_s = 44.1$ kHz

number of bits per sample, $n = 16$ bits

So, the resulting bit rate of the sampled signal is

$$R_b = nf_s$$
$$= 16 \times 44.1 \text{ kHz}$$
$$= 705.6 \text{ kbits/sec}$$

So, the resulting number of bits for the music with duration 50 minutes is

$$\text{Number of bits} = (50 \times 60) \times R_b$$
$$= 3000 \times 705.6 \times 10^3$$

$$= 2.1168 \times 10^9 \text{ bits}$$

This is the resulting number of bits per channel (left and right). Since, the two audio signals from the left (L) and right (R) microphones in a recording studio or a concert hall are sampled and digitized, so the overall number of bits is obtained as

$$\text{Number of bits} = 2 \times 2.1168 \times 10^9$$
$$= 4.2336 \times 10^9 \text{ bits}$$
$$= 4233.6 \text{ Mbits}$$

SOL 6.2.27 Correct answer is 256.

Given, the bandwidth of each of the four voice signals,

$$f_{m_o} = 4 \text{ kHz}$$

Number of quantization levels, $q = 256$

Since, the voice signals are sampled at Nyquist rate, so we have the sampling frequency for each of the signal as

$$f_{s0} = 2f_m = 2 \times 4 \text{ kHz}$$
$$= 8 \text{ kHz}$$

Therefore, the number of pulses per second (frequency) of the multiplexed signal is

$$f_s = 4 \times f_{s0}$$
$$= 4 \times 8 = 32 \text{ kHz}$$

So, the bit transmission rate of the time division multiplexed signal is

$$R_b = nf_s \qquad \text{...(1)}$$

where n is the number of bits per sample defined as

$$q = 2^n$$

So, $n = \log_2 256 = 8$

Substituting it in equation (1), we get the bit rate as

$$R_b = 8 \times 32 \text{ kHz}$$
$$= 256 \text{ kbits/sec}$$

SOL 6.2.28 Correct answer is 180.

Given, the maximum frequency of the analog data, $f_m = 30$ kHz

The quantization level (digitization level), $q = 6$

So, the required number of bits per sample for digitization is given by

$$2^n \geq 6$$

or, $n = 3$ bits

Now, the sampling frequency for the analog data is given by Nyquist rate as

$$f_s = 2f_m$$
$$= 2 \times 30$$
$$= 60 \text{ kHz}$$

Therefore, the rate of generated digital signal is

$$R_b = nf_s$$
$$= 3 \times 60 \text{ kHz}$$
$$= 180 \text{ kbits/sec}$$

SOL 6.2.29 Correct answer is 32.

Given, the number of quantization levels, $q = 16$

maximum signal frequency, $f_m = 4$ kHz

So, the sampling frequency for the message signal is given by its Nyquist rate as

$$f_s = 2f_m$$
$$= 2 \times 4 = 8 \text{ kHz}$$

Again, the number of bits per sample is obtained as

$$2^n \geq q$$

or, $\quad 2^n \geq 16$

or $\quad n = 4$

Therefore, the bit transmission rate is given by

$$R_b = nf_s$$
$$= 4 \times 8$$
$$= 32 \text{ kbits/sec}$$

SOL 6.2.30 Correct answer is 64.

Given the frequency range of the speech signal

$$300 \text{ Hz} < f < 3 \text{ kHz}$$

So, the speech signal is band limited to $f_m = 3$ kHz

The sampling frequency of the signal must be greater than Nyquist rate

$$f_s \geq 2f_m = 2 \times 3 = 6 \text{ kHz}$$

Here, the sampling frequency is

$$f_s = 8 \text{ kHz}$$

This satisfies the Nyquist criterion. Since, the number of quantization levels is

$$q = 256$$

So, the number of bits per sample is

$$n = \log_2 256 = 8$$

Therefore, the output bit rate is given by

$$R_b = nf_s = 8 \times 8 = 64 \text{ kbps}$$

SOL 6.2.31 Correct answer is 128.

Given, the transmission bandwidth over the channel

$$B_T = 1 \text{ MHz}$$

This is the bandwidth at output of PSK modulator having roll factor

$$\alpha = 25\% = 0.25$$

So, we obtain the overall system bandwidth at time division multiplexer output as

$$B_{\text{overall}} = \frac{B_T}{1+\alpha}$$
$$= \frac{10^6}{1+0.25} = 8 \times 10^5 \text{ Hz}$$
$$= 800 \text{ kbits/ sec}$$

Since, the number of pulses from data source is 240 k bits/sec. So, we obtain the bit rate at the output of binary encoder as

$$R_b = 800 \text{ k bits/s} - 240 \text{ k bits/ sec}$$
$$= 560 \text{ kbits/ sec}$$

Now, sampler has the sampling frequency of 8 kHz. Since, there are 10 voice signals applied to the sampler. So, we have the overall sampling frequency,

$$f_s = 10 \times 8$$

$$= 80 \text{ kHz}$$
$$= 8 \times 10^4 \text{ samples/second}$$

Therefore, the number of bits per sample is given by

$$n = \frac{R_b}{f_s} = \frac{560 \text{ k bits/ sec}}{8 \times 10^4 \text{ samples/ sec}}$$
$$= 7 \text{ bits/sample}$$

Thus, number of quantization level is given by

$$q = 2^n = 2^7 = 128 \text{ levels}$$

SOL 6.2.32 Correct answer is 83.

The allowable voice bandwidth for telephone line transmission,

$$W = 3 \text{ kHz} = 3000 \text{ samples/sec}$$

The number of quantization level,

$$q = 16$$

Minimum interval allowed for reliable identification of a bit,

$$T_{\text{bit}} = 1\,\mu\text{s}$$

So, we have the maximum allowable bit rate as

$$R_{b,\max} = \frac{1}{T_{\text{bit}}} = \frac{1}{1\,\mu\text{s}} = 10^6 \text{ bits/ sec}$$

Also, we get the number of bits per sample,

$$n = \log_2 q = \log_2 16 = 4 \text{ bits/sample}$$

Now, let the number of voice signals that can be multiplexed be M so, we have over all sampling frequency for the communication channel as

$$f_s = MW = M \times 3000$$
$$= 3000\,N \text{ samples/sec}$$

Therefore, the bit rate at the receiving end is given by

$$R_b = nf_s$$
$$= (4 \text{ bits/sample}) \times (3000M \text{ samples/ sec})$$
$$= 12000M \text{ bits/ sec}$$

This bit rate must be less than the maximum bit rate, i.e.

$$R_b < R_{b,\max}$$
$$12000M < 10^6$$

or $$M < \frac{10^6}{12000} = 83.33$$

or $$M \approx 83 \text{ signals}$$

SOL 6.2.33 Correct answer is 3.125.

Given, the number of quantization level, $q = 256$

Bandwidth of three input signals,

$$W_1 = 5 \text{ kHz},\ W_2 = 10 \text{ kHz},\ W_3 = 5 \text{ kHz}$$

So, we have the number of bits per sample as

$$n = \log_2 q = \log_2 256 = 8$$

Since, the bandwidth of signals are harmonically related, we have the overall bandwidth of multiplexed signals as

$$W = W_1 + W_2 + W_3$$
$$= 5 + 10 + 5 = 20 \text{ kHz}$$

So, the Nyquist rate for multiplexed signal is given by

$$f_N = 2W = 2 \times 20 = 40 \text{ kHz}$$

This is the sampling frequency for the transmitted signal. Therefore, the bit rate of the transmitted signal is obtained as

$$\begin{aligned} R_b &= nf_s \\ &= 8 \text{ bits/sample} \times 40 \times 10^3 \text{ samples/ sec} \\ &= 320 \text{ k bits/ sec} \end{aligned}$$

Thus, the maximum bit duration for the transmitted signal is

$$\begin{aligned} T_{\text{bit}} &= \frac{1}{R_b} \\ &= \frac{1}{320 \times 10^3} \\ &= 3.125 \times 10^{-6} \text{ sec} = 3.125\,\mu\text{sec} \end{aligned}$$

SOL 6.2.34 Correct answer is 160.

We have the bit rate of transmitted signal,

$$R_b = 320 \text{ k bits/ sec}$$

So, the minimum channel bandwidth required to pass the PCM signal is given by

$$\begin{aligned} B &= \frac{R_b}{2} \\ &= \frac{320}{2} = 160 \text{ kHz} \end{aligned}$$

SOL 6.2.35 Correct answer is 112.

Given, the time period of transmitting signal,

$$T = 71.4\,\mu\text{s} = 71.4 \times 10^{-6} \text{ sec}$$

So, we have frequency of the signal as

$$f = \frac{1}{T} = \frac{1}{71.4 \times 10^{-6}} = 1.4 \times 10^4 \text{ Hz}$$

Therefore, the 4^{th} harmonic of the signal is given by

$$\begin{aligned} f_{4^{\text{th}}\text{ harmonic}} &= 4 \times 1.4 \times 10^4 \\ &= 5.6 \times 10^4 \text{ Hz} \end{aligned}$$

This must be the bandwidth of the signal, i.e.

$$W = f_{4^{\text{th}}\text{harmonic}} = 5.6 \times 10^4 \text{ Hz}$$

Thus, the minimum sampling rate (Nyquist rate) for the signal is

$$\begin{aligned} f_s &= 2W = 2 \times (5.6 \times 10^4) \\ &= 11.2 \times 10^4 \text{ Hz} = 112 \text{ kHz} \end{aligned}$$

SOL 6.2.36 Correct answer is 732.5.

Given, the number of bits representing analog signal

$$n = 14 \text{ bits}$$

Peak to peak voltage range,

$$2m_p = 6 - (-6) = 12 \text{ V}$$

So, we have the number of digitzation (quantization) level,

$$q = 2^n = 2^{14} = 16384$$

Thus, the resolution of digitization is given by

$$\begin{aligned} \text{Resolution} &= \frac{12}{q-1} \\ &= \frac{12}{16384 - 1} = 7.3246 \times 10^{-4} \text{ V} \\ &= 732.5\,\mu\text{V} \end{aligned}$$

SOL 6.2.37 Correct answer is 3.

Given, the maximum voltage range $V_{max} = 1\text{ volt}$

Input voltage of compander, $V_{in} = 0.25\text{ volt}$

For μ-law compander, $\mu = 255$

So, the output voltage of compander is given by

$$V_{out} = \frac{V_{max}\ln\left[1+\frac{\mu V_{in}}{V_{max}}\right]}{\ln[1+\mu]}$$

$$= \frac{1\ln\left[1+\frac{255\times 0.25}{1}\right]}{\ln[1+255]}$$

$$= 0.752\text{ volt}$$

Therefore, the gain of compander is obtained as

$$G = \frac{V_{out}}{V_{in}} = \frac{0.752}{0.25} = 3$$

SOL 6.2.38 Correct answer is 1200.

Given, the data rate of communication system

$$R_b = 9600\text{ bits/sec}$$

Number of bits encoded into each level,

$$k = 4\text{ bits/level}$$

So, we obtain the baud rate for the system,

$$D = \frac{R_b}{k} = \frac{9600\text{ bits/sec}}{4\text{ bits/level}}$$

$$= 2400\text{ levels/sec}$$

Therefore, the minimum required bandwidth for the channel is

$$B = \frac{D}{2} = \frac{2400}{2} = 1200\text{ Hz}$$

SOL 6.2.39 Correct answer is 1.33.

Given, the number of levels for polar NRZ line code, $M = 4$

The bandwidth of channel, $B = 4\text{ kHz}$

Roll off factor, $\alpha = 0.5$

So, we get the number of bits per level,

$$k = \log_2 M = \log_2 4 = 2\text{ bits/symbol}$$

Therefore, the bit rate of PCM signal is obtained as

$$R_b = kD$$

$$= (2\text{ bits/symbol})\times\left(\frac{2B}{1+\alpha}\right)$$

$$= 2\times\frac{2\times 4\text{ kHz}}{1+0.5}$$

$$= 10.67\text{ kHz}$$

Again, we have the number of quantization level for PCM signal as $q = 16$.

So, the number of bits per sample for the PCM signal is given by

$$n = \log_2 q = \log_2 16 = 4$$

Therefore, the sampling frequency of the signal is given by

$$f_s = \frac{R_b}{n}$$

$$= \frac{10.67\text{ kHz}}{4} = 2.67\text{ k samples/sec}$$

Thus, the maximum bandwidth analog signal is

$$W = \frac{f_s}{2}$$
$$= \frac{2.67\text{ kHz}}{2} = 1.33\text{ kHz}$$

SOL 6.2.40 Correct answer is 61.2.

Given, the bandwidth of audio signal, $W = 3400$ Hz

For compander, we have $\mu = 255$

For the μ-law compander, we have signal to noise ratio

$$\left(\frac{S}{N}\right)_{\text{dB}} = 6.02n + 4.77 - 20\log[\ln(1+\mu)]$$

This value must be at least 40 dB, so we have

$$40 \le 6.02n + 4.77 - 20\log[\ln(1+255)]$$
$$n \ge \frac{40 - 4.77 + 20\log[\ln(256)]}{6.02}$$
$$= 8.32\text{ bits}$$

So, we use $n = 9$. Therefore bit rate is obtained as

$$R_b = nf_s$$
$$= 9 \times (2W)$$
$$= 9 \times 2 \times 3400$$
$$= 6.12 \times 10^4\text{ bits/ sec}$$
$$= 61.2\text{ k bits/ sec}$$

SOL 6.2.41 Correct answer is 7.

Given the error at receiving end,

$$|\text{error}| \le 0.5\%\text{ of peak-to-peak full scale value}$$

So, we have

$$\frac{1}{2}\left(\frac{2m_p}{2^n}\right) \le \frac{0.5}{100} \times 2m_p$$

or, $$2^n \ge 100$$

or, $$n \ge \log_2 100 = 6.65$$

Therefore, the minimum number of bits per sample (word) is

$$n = 7\text{ bits/word}$$

SOL 6.2.42 Correct answer is 5600.

Given the message signal,

$$m(t) = 4\sin 2\pi(10)t + 5\sin 2\pi(20)t$$

Also, we have the step size,

$$\delta = 0.05\pi$$

To avoid the slope overload, we have the required condition

$$\delta f_s \ge \max\left|\frac{dm(t)}{dt}\right|$$
$$0.05\pi f_s \ge \max\{80\pi\cos 2\pi(10t) + 200\pi\cos 2\pi(20)t\}$$
$$0.05\pi f_s \ge 280\pi$$
$$f_s \ge \frac{280\pi}{0.05\pi} = 5600\text{ Hz}$$

This is the minimum required sampling frequency.

SOL 6.2.43 Correct answer is 3600.

We have the frequency components in the message signal as

$$f_1 = 500 \text{ Hz}$$
$$f_2 = 750 \text{ Hz}$$
$$f_3 = 1800 \text{ Hz}$$

So, the message signal is bandlimited to

$$W = f_3 = 1800 \text{ Hz}$$

Therefore, the sampling frequency of the signal is obtained as

$$f_s \geq 2W = 2 \times 1800$$
$$= 3600 \text{ Hz}$$

SOL 6.2.44 Correct answer is 20.

Since, the message signal is bandlimited to

$$W = 100 \text{ kHz}$$

So, we have the Nyquist rate for the signal as

$$f_N = 2W = 2 \times 100 = 200 \text{ kHz}$$

Since, the signal is sampled at 90% of its Nyquist rate, so we have the sampling frequency of the signal as

$$f_s = \frac{90}{100} f_N$$
$$= 0.9 \times 200 = 180 \text{ kHz}$$

Therefore, the signal to distortion ratio (SDR) is obtained as

$$\text{SDR} = \frac{\int_0^{f_s/2} S(f)df}{\int_{f_s/2}^{W} S(f)df} = \frac{\int_0^{90 \times 10^3} (1 - 10^{-5} f)df}{\int_{90 \times 10^3}^{10^5} (1 - 10^{-5} f)df}$$

$$= \frac{\left[f - \frac{10^{-5} f^2}{2}\right]_0^{90 \times 10^3}}{\left[f - \frac{10^{-5} f^2}{2}\right]_{90 \times 10^3}^{10^5}}$$

$$= \frac{90 \times 10^3 - \frac{10^{-5} \times (90 \times 10^3)^2}{2}}{10^5 - \frac{10^{-5} \times (10^5)^2}{2} - 90 \times 10^3 + \frac{10^{-5}(90 \times 10^3)^2}{2}}$$

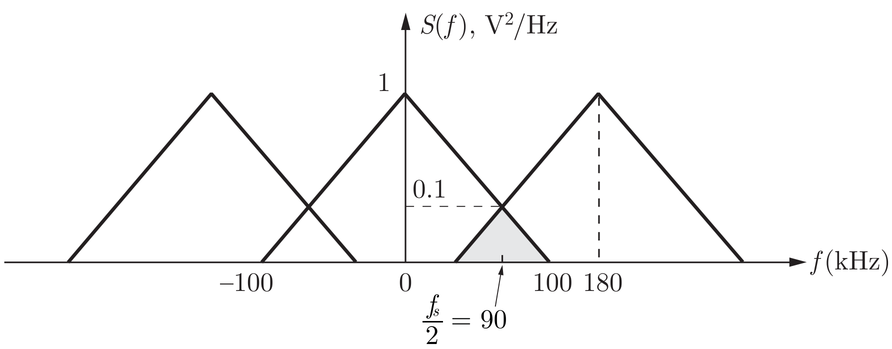
$$= 99 \text{ or } 20 \text{ dB}$$

ALTERNATIVE METHOD :

We have the sampling frequency,

$$f_s = 180 \text{ kHz}$$

If the message signal is sampled at this frequency, we get the frequency spectrum of sampled waveform as shown below.



Consider the sampled signal in the range

$$0 < f < 180 \text{ kHz}$$

For this region, we have the undistorted message signal power as

$$P_{so} = \text{Area of unshaded region}$$

$$= 2 \times \left[\frac{1}{2} \times 100 \times 1\right] - \frac{1}{2} \times 20 \times 0.1$$

$$= 100 - 1 = 99$$

Also, the power in distorted signal is given by

$$P_{do} = \text{Area of shaded region}$$

$$= \frac{1}{2} \times 20 \times 0.1 = 1$$

So, we get the signal to distortion ratio as

$$\text{SDR} = \frac{P_{so}}{P_{do}} = \frac{99}{1} = 99 \text{ or } 20 \text{ dB}$$

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SOLUTIONS 6.3

SOL 6.3.1 Option (C) is correct.
The required sampling rate for no aliasing in a PCM system is given by

$$f_s \geq 2f_m$$

where f_m is the input message frequency. So, aliasing occurs when a signal is mistakenly sampled at less than twice the input frequency.

SOL 6.3.2 Option (B) is correct.

SOL 6.3.3 Option (D) is correct.

SOL 6.3.4 Option (B) is correct.

SOL 6.3.5 Option (D) is correct.

SOL 6.3.6 Option (C) is correct.

SOL 6.3.7 Option (C) is correct.

SOL 6.3.8 Option (A) is correct.

SOL 6.3.9 Option (D) is correct.

SOL 6.3.10 Option (A) is correct.
In flat-top sampling, the aperture effect occurs due to application of rectangular pulse. It causes an amplitude distortion and time delay.

SOL 6.3.11 Option (B) is correct.
In PCM, the quantization noise power is defined as

$$\overline{\varepsilon^2} = \frac{\delta^2}{12}$$

where δ is the step size of the quantization given by

$$\delta = \frac{2m_p}{q}$$

where $2m_p$ is the peak to peak amplitude and q is the number of quantization level. Thus, the quantization noise depends on number of quantization levels.

SOL 6.3.12 Option (C) is correct.
Pulse modulation can be either analog or digital. Some typical analog and digital pulse modulations are categorized below.
Analog pulse modulation :
(i) Pulse amplitude modulation (PAM)
(ii) Pulse width modulation (PWM)
(iii) Pulse position modulation (PPM)
Digital pulse modulation :
(i) Delta modulation (DM)
(ii) Pulse code modulation (PCM)
(iii) Differential pulse code modulation (DPCM)

SOL 6.3.13 Option (D) is correct.

SOL 6.3.14 Option (C) is correct.

SOL 6.3.15 Option (D) is correct.

SOL 6.3.16 Option (C) is correct.

SOL 6.3.17 Option (A) is correct.

SOL 6.3.18 Option (D) is correct.

SOL 6.3.19 Option (D) is correct.
Analog pulse modulation results when some attributes of a pulse varies continuously in one-to-one correspondence with a sample value.
The attributes may be the amplitude, width, or position which lead to the following modulations:
(i) Pulse amplitude modulation (PAM)
(ii) Pulse width modulation (PWM)
(iii) Pulse position modulation (PPM)

SOL 6.3.20 Option (C) is correct.

SOL 6.3.21 Option (B) is correct.

SOL 6.3.22 Option (B) is correct.

SOL 6.3.23 Option (C) is correct.

SOL 6.3.24 Option (B) is correct.

SOL 6.3.25 Option (C) is correct.

SOL 6.3.26 Option (B) is correct.

SOL 6.3.27 Option (C) is correct.

SOL 6.3.28 Option (B) is correct.

SOL 6.3.29 Option (C) is correct.

SOL 6.3.30 Option (C) is correct.

SOL 6.3.31 Option (C) is correct.

SOL 6.3.32 Option (D) is correct.

SOL 6.3.33 Option (A) is correct.

SOL 6.3.34 Option (A) is correct.

SOL 6.3.35 Option (C) is correct.

SOL 6.3.36 Option (A) is correct.

SOL 6.3.37 Option (B) is correct.
The transmission bandwidth must have a value such that it passes through the channel without any attenuation.

Thus, we have

$$W \leq W_{\text{channel}}$$

where W is transmission bandwidth and W_{channel} is channel bandwidth.

SOL 6.3.38 Option (B) is correct.
According to Nyquist theorem, the minimum sampling frequency required to ensure no loss in information is given by

$$f_s = 2f_m$$

SOL 6.3.39 Option (D) is correct.

SOL 6.3.40 Option (B) is correct.
The number of bits per sample in a PCM system is given by

$$n = \log_2 q$$

In PCM, each of n bits is represented by a pulse. So, the number of pulses in a code group is given by

$$p = \log_2 q$$

SOL 6.3.41 Option (B) is correct.

SOL 6.3.42 Option (D) is correct.

SOL 6.3.43 Option (C) is correct.

SOL 6.3.44 Option (C) is correct.
In a PCM system there are two error sources:
(i) Quantization noise
(ii) Channel noise
Channel noise in the system is independent of quantization noise. So, with increase in quantization noise, channel noise will remain unaffected.

SOL 6.3.45 Option (D) is correct.

SOL 6.3.46 Option (C) is correct.

SOL 6.3.47 Option (B) is correct.

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